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Near-Brane SU(6)-Stronglycoupled-Origin Exotic Higgses and Gauge-like Scalars in Scherk-Schwarz Breaking of 5-Dimensional SU(6) via Double Vacua

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Abstract

The symmetry breaking of 5-dimensional SU(6) is realized by Scherk-Schwarz mechanism through trivial and pseudo non-trivial manner with orbifold S^1/Z_2 breakings to produce dimensional deconstruction 5D SU(6)→4D SU(6). The later also induces near-brane strongly-coupled SU(6) will-be-SimplestLittleHiggs scalar to further break the symmetry into $\mathrm{SU(3)_c \otimes SU(3)_H \otimes U(1)_C}$ under triplet-triplet splitting as required by trivial-and-pseudonontrivial conditions. The model successfully provides a scenario of the origin of collective breaking and one-by-one breaking patterns where the first comprises of both shift and asymptotic shift symmetry accompanied by local gauge symmetry breaking, and the later involves only the shift symmetry breaking. Heisenberg scalar, a basic constituent of exotic Higgs and gauge-like scalar, emerges from the correspondent asymptotic shift and local symmetry breaking and becomes a gauge-like single scalar while other exotic scalars from collective breaking. The role of Nambu-Goldstone boson (NGB) in relation to Higgs in the so-called swallowing-digesting process to form a Heisenberg scalar is established where double vacua system, underlied by local-global correspondence and triplet-triplet splitting, facilitates the formation of exotic scalars which have the most preferred range of mass for 3-scalar Higgs and almost a continuous range of mass up to 1.5 TeV for Heisenberg scalar, 3-component pseudo (Higgs-like) Heisenberg scalar and gauge-like scalar.

Keywords: Orbifold, Scherk-Schwarz breaking, Little Higgs, Standard Model, Uncertainty principles, Nambu-Goldstone boson

1 Introduction

GUT model based on SU(6) symmetry [1] suggests that the electroweak scale physics is realized via symmetry breaking $\mathrm{SU(6)}\to\mathrm{SU(3)_c}\otimes\mathrm{SU(3)_H}\otimes\mathrm{U(1)_C}$ and subsequently to $\mathrm{SU(3)_H}\to\mathrm{SU(2)_L}\otimes\mathrm{U(1)_B}$ but has a severe shortcoming, that is, no appropriate Higgs multiplet [2].

Following recent development on extra dimension physics, the Scherk-Schwarz breaking [21,22,30] and Orbifold breaking [26,30] have opened the way for producing Higgs through the extra-dimensional compactification effect which induces the Higgs itself, and known as the gauge-Higgs boson unification [13,14,30]. Recently a grand gauge-Higgs unification based on 5D SU(6) compactified on an orbifold S^1/Z_2 with fermions in two 6-plet and one 20-plet shows successful breaking with no proton decay at tree level but a little low compactification scale and heavy Higgs [10]. Other work such as Hosotani mechanism with non-zero VEV being developed by extra-dimensional parameters results in instant gauge symmetry breaking after dimensional deconstruction [28,49] and some other works in extra dimension and non-Higgs mechanism have been developed [3,13,14,15,26,40,45] including the pioneer one [35].

On the other hand Little Higgs mechanism, with pseudo Nambu-Goldstone boson (PNB) as Little Higgs, has been developed simultaneously. It provides massive PNB after shift symmetry breaking [33,36,37,38,41,42,48] and becomes an alternative to Higgs breaking.

Corresponding to 5D SU(6) gauge symmetry, based on AdS/CFT correspondence, SU(6) global symmetry also exists [3,10], and breaks after shift symmetry breaking takes place, and produces massive PNB. It means that the scalars come from the fifth components of 5D gauge bosons [10,13,14,30] and/or directly from the bulk [15].

In the paper [3], special conditions of Scherk-Schwarz mechanism are utilized to resolve the problem of breaking the SU(6) GUT. The trivial and the non-trivial breaking pattern are simultaneously realized by compactification of orbifold S^1/Z_2 in 5-dimensional (5D) SU(6), not like the trivial the (pseudo) non-trivial condition generates the scalar bosons. The condition facilitates for the periodic 5D scalar [3,13,14,15,26] with extra-dimensional global symmetry for small extra dimension in the so-called near-brane area [3,10]. Here, in the near-brane area, the first symmetry breaking of 5D SU(6)→ 4D SU(6) is triggered by Scherk-Schwarz mechanism and followed by trivial and pseudo non-trivial Orbifold breaking [3,26,28,30] to produce SU(6)-origin would-be Little (Baby) Higgs scalar as the origin of SU(6) will-be-SimplestLittle Higgs and SU(6) Baby Higgs scalars successively [3]. Trivial Orbifold Breaking (TOB) and Pseudo non-trivial Orbifold Breaking (POB) which facilitate dimensional deconstruction but still keep the symmetry intact, in principle, do produce 'exact' scalar boson, and the perturbative, incomplete series of exponential form, of strongly-coupled SU(6)-origin would-be Little-like (Baby) Higgses. It means that SU(6)-origin would-be Little-like (Baby) Higgses decouple from and cannot exist in SU(6) and be transformed into SU(6) will-be-SimplestLittleHiggs scalar which transforms itself again under triplet-triplet splitting into Simplest-like Little Higgs [3,50].

In this paper the 5D near-brane model is established with a special local-global gauge correspondence where a generalized Coleman-Weinberg potential is defined accordingly, and followed by triplet-triplet splitting of the SU(6) will-be-SimplestLittleHiggs scalar sextet. Nevertheless a basic discussion on the splitting potential and the emerging scalars are given afterward where the breaking pattern i.e. one-by-one and collective are briefly discussed, together with the masses of the emerging PNB Higgses and gauge-like scalars. Next, one discusses the NGBs in double vacua system [39] which becomes one of key issues in understanding exotic Higgses and gauge-like scalars. It starts with Heisenberg scalar, a basic constituent of exotic scalar above, the role of NGB in its formation and the swallowing-digesting process, instead of eating NGB [5,19,47]. Two kinds of Heisenberg scalars emerge, its contributions to exotic Higgses or gauge-like scalars are discussed elaborately. A short look is dedicated to shift and asymptotic shift symmetry, the controlling conditions for the breakings, the NGBs allocation in the formation of exotic scalars, the emerging and masses of gauge (Higgs)-like 3-component (scalar) scalar (Higgs).

Finally, mass generation in the pseudo Heisenberg scalar is provided, while the breaking patterns of shift and asymptotic shift symmetry can be found in [5]. Before closing the discussion some phenomenological aspects are reviewed and given in terms of some observables, followed by the conclusion.

2 The 5D Near-brane Model with SU(6) Scalar

2.1 The Scherk-Schwarz and Orbifold breaking

First of all let us consider the orbifold breaking in 5D SU(6) compactified in $\mathcal{M}4 \times S^1/Z_2$. Before discussing the details, a brief review on Scherk-Schwarz mechanism on orbifold S^1/Z_2 is given below.

The invariance of a theory compactified on 5-dimensional space, $\mathcal{M}4 \times S^1/Z_2$, demands $\mathcal{L}_5[\phi(x,y)] = \mathcal{L}_5[\phi(x,\tau_g(y))]$. The ordinary compactification satisfies $\phi(x,\tau_g(y)) = \phi(x,y)$ which is a special case of general Scherk-Schwarz compactification condition $\phi(x,\tau_g(y)) = T_g\phi(x,y)$[26, 28, 30]. Here, $\tau_g(y)$ is the mapping operator for y, and T_g is the twist transformation operator.

In general, orbifold compactifications have similar principles. The condition is written as $\phi(x,\zeta_2(y)) = Z_2\phi(x,y)$. The operator T_g should satisfy the so-called consistency condition as below, anyway,

$$T_g Z_2 T_g = Z_2, \quad T_g = e^{2i\pi\vec{\beta}\cdot\vec{\lambda}} = e^{2i\pi\omega Q} \tag{1}$$

where $\lambda^{a'}$ are the hermitian generators and Q is the generator with a predefined direction in generator space, while ω and $\beta^{a'}$ are the corresponding parameters. Combining with the above consistency condition and expanding infinitesimally one immediately finds the condition [26,28],

$$\left\{\vec{\beta}\cdot\vec{\lambda}, Z_2\right\} = 0 \quad \text{and} \quad [T_g, Z_2] = 0 \tag{2}$$

These relations determine the broken and unbroken parts of the generators under consideration. The latter also gives the singular solution $T_g = \pm 1$.

For 5D theory compactified on the S^1/Z_2 orbifold with the Scherk-Schwarz twist as in Eq. (1), the twisted field obeys,

$$\phi(x, y + 2\pi R) = e^{2i\pi\omega Q}\phi(x,y) \tag{3}$$

where R is the compactification radius. Symmetry breaking is achieved if the symmetry generated by Q is broken by the 5D kinetic term and $Q'(Q)$ satisfies the (anti)commutative relation in Eq. (2) [26,28], that is

$$\{\omega Q, Z_2\} = \omega\{Q, Z_2\} = 0, \quad [Q', Z_2] = 0 \tag{4}$$

On the other hand, the unbroken parts generated by Q' are determined by the second relation in Eq. (2) respectively.

Now we are ready to apply the preceding discussion on the S^1/Z_2 orbifold to SU(6) [22,26,30]. Z_2 for SU(6) can be constructed based on 3 arrays of SU(2) type matrix along its diagonal elements as follow,

$$Z_2 = \begin{pmatrix} 1 & 0 & & & & \\ 0 & 1 & & & & \\ & & 1 & 0 & & \\ & & 0 & -1 & & \\ & & & & -1 & 0 \\ & & & & 0 & -1 \end{pmatrix} \tag{5}$$

This form satisfies the boundary conditions of S^1/Z_2 orbifold suitable to realize the symmetry breaking SU(6)→SU(3)⊗SU(3)⊗U(1) in the non-trivial pattern.

Let us adopt the special condition in ii) and iii) of Appendix A (A.1) for the current 5D → 4D case of SU(6) namely trivial and pseudo non-trivial patterns and write as 5D SU(6)→4D SU(6) or 5D ($y \sim 0$) SU(6) from which a near-brane is to be defined later. In the near-brane one has the further breaking as

$$\text{4D SU(6)} \to \text{4D SU(3)} \times \text{SU(3)} \times \text{U(1)} \tag{6}$$

where 4D here is equivalent to 5D ($y \sim 0$) and SU(3)×SU(3)×U(1) with $y = 0$ is brane itself.

2.2 Near-brane Coleman-Weinberg potential and global-local gauge correspondence

We adopt the 5D-model in near-brane where extra dimension $y \sim 0$ and SU(6) global gauge symmetry exist (to be explained) with the 4D particles living in the 2 branes and 5D gauge bosons as well as scalar bosons in the bulk. One brane corresponds to fixed point $y = 0$ and the other brane corresponds to another fixed point $y = \pi R$ of the S^1/Z_2 orbifold.

The Lagrangian can be written accordingly as

$$\mathcal{L}_5^{\text{SU(6)}} = D^M \Phi^\dagger D_M \Phi, \quad M = (\mu, y), \tag{7}$$

where $\Phi = (\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6)^T \equiv [\Phi_k], k = 1, 2, \cdots, 6$ is scalar boson in the fundamental representation of SU(6), and scalar field Φ is expressed as periodic scalar field $\widetilde{\Phi}$ via the following relationship [6]

$$\Phi(x, y) = e^{i\omega Q_v y/R}\, \widetilde{\Phi}(x, y) = e^{iQ_v \alpha}\, \widetilde{\Phi}(x, y), \tag{8}$$

which can be obtained as solution of Eq. (3), α turns out to be a global gauge phase factor for $\alpha \ll 1$ and to be discussed further later, and Q_v represents SU(6) broken generators in the direction of VEVs [13,14,30]. Defining $D^\mu(D_\mu)$ as 4D-covariant derivative and $D^y(D_y)$ as fifth-dimensional covariant derivative with $T^a = \lambda^a/2(= T_a)$, and g_5 is the 5D coupling constant

$$\begin{aligned} D_\mu &= \partial_\mu - ig_5 A^a_\mu T_a, \quad D^\mu = \partial^\mu + ig_5 A^\mu_a T^a \quad \text{and} \\ D_y &= \partial_y + ig_5 A^a_y T_a, \quad D^y = \partial^y - ig_5 A^y_a T^a, \end{aligned} \tag{9}$$

one can separate the 4D-brane from the bulk Lagrangian $\mathcal{L}_5^{\text{SU(6)}} = \mathcal{L}_\mu^{\text{brane}} + \mathcal{L}_{(\theta y),y}^{\text{near-brane}} + \mathcal{L}_y^{\text{SU(6)}}$, where 4D near-brane is just in-between the brane and the bulk.

In the brane $y = 0$ and near-brane $y \sim 0$ the even scalar bosons in Eq. (8) becomes as

$$\widetilde{\Phi}^{(i)}(x) = \widetilde{\Phi}^{(i)}(x, y) \mid_{y=0 \text{ or } \sim 0}, i = 1, 2. \tag{10}$$

For the upper-near-brane space Neumann boundary condition dictates $D^y \widetilde{\Phi}^\dagger = D_y \widetilde{\Phi} = 0$ and the property of extra-dimensional dominance $D^\mu \widetilde{\Phi}^\dagger = D_\mu \widetilde{\Phi} = 0$ which make the only upper-near-brane equation with $\delta(y) = 1$ for $y \sim 0$ and $Q_v = 0$, as follows [3],

$$\mathcal{L}_y^{\text{near-brane}} = \frac{1}{2} g_5^2 \left(\widetilde{\Phi}^{(i)\dagger} A_a^y \right) \left(A_y^a \widetilde{\Phi}^{(i)} \right), \tag{11}$$

where now $\widetilde{\Phi}^{(i)} = \widetilde{\Phi}_+^{(i)}(x)$ and $\widetilde{\Phi}^{(i)\dagger} = \widetilde{\Phi}_+^{(i)\dagger}(x)$, with $i = 1, 2$, and $(+)$ denoting even parity. In this upper-near-brane bulk (y-area), one has the subsets (sextet out of 2×9 broken $A_{\hat{a}}^y$ and $A_y^{\hat{a}}$) as

$$A_{\hat{a}}^y T^{\hat{a}} \supset \widetilde{\Phi}^{(j)}, \quad A_y^{\hat{a}} T_{\hat{a}} \supset \widetilde{\Phi}^{(j)\dagger}, \tag{12}$$

which is based on gauge-Higgs unification principle, where $\widetilde{\Phi}^{(j)}(\text{or } \widetilde{\Phi}^{(j)\dagger})$ is diagonal 3×3 sub-matrix component of 6×6 matrix of $A_{\hat{a}}^y T^{\hat{a}}\,(\text{or } A_y^{\hat{a}} T_{\hat{a}})$ and $j = 1, 2$ as in Eq. (12). Consequently Eq. (11) can be rewritten, using $\widetilde{\Phi}^{(j)}$ as diagonal component of $A_{\hat{a}}^y T^{\hat{a}}$, and $\lambda_y^{(6)} = g_5^2$, as [3],

$$V_y^{(6)} = \lambda_y^{(6)} (\widetilde{\Phi}^{(i)\dagger} \widetilde{\Phi}^{(j)}) (\widetilde{\Phi}^{(j)\dagger} \widetilde{\Phi}^{(i)}). \tag{13}$$

In order to determine $\widetilde{\Phi}^{(i)}$ and $\widetilde{\Phi}^{(j)}$ which are not necessarily the same but it must be corresponding one to another due to existing correspondence between 5D local gauge and 4D global gauge symmetry as shown later on, then Eq. (8) can be written, applying $y \sim 0$ for periodic scalar under global gauge transformation $e^{iQ_v \alpha}$, as [3],

$$\Phi(x, 0) \sim e^{iQ_v \alpha} \widetilde{\Phi}(x), \tag{14}$$

where global gauge is defined as,

$$\alpha = \frac{\omega y}{R}, \tag{15}$$

with ω is Scherk-Schwarz parameter which is dynamical now due to correspondence property below. Recalling local gauge change $\Delta\alpha(x) \sim \alpha'(x)\Delta x$ for $y \sim 0$ and setting the ansatz, a particle moving from bulk (near-brane) following the uncertainty principle $\Delta x\, \alpha\, \frac{1}{\Delta p}$, to near-brane (bulk) experiences momentum and position uncertainty. One finally finds, defining $\alpha'(x) = \frac{y}{R}$ as a constant, as follows [3,43,44],

$$\Delta\alpha(x) \sim \frac{\omega y}{R} = \alpha, \quad \omega(\kappa, \lambda) = \frac{\lambda}{\kappa^{(1+\epsilon)}}, \quad \epsilon > 0, \tag{16}$$

where $\kappa = \Delta p$ and λ twist per unit length of field, clearly there is a correspondence between $\Delta\alpha(x)$ and global gauge α. This allows Eq. (13) to consist of Higgs (pseudo Nambu-Goldstone boson, PNB) and gauge-like scalar as its correspondence. To start our analysis some parameters must be defined accordingly.

At the orbifold singular points which are 4D and have the twist factor $T_g = +1$ for $y = 0$ and $T_g = -1$ for $y = \pi R$, one can assign without loss of generality, two non-zero VEVs at one fixed point such as [3],

$$v = \frac{1}{\sqrt{\pi R}} \begin{pmatrix} 0 \\ 0 \\ f_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, v' = \frac{1}{\sqrt{\pi R}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ f_2 \end{pmatrix}, \text{for } y = 0. \tag{17}$$

The parametrization of SU(6) would-be Baby (Little) Higgs is governed by the number of scalar doublets which are allowed to be put in 6×6 matrix. Thus it depends on the number of the generated NGBs through the condition [3],

$$a'_{jk}\widetilde{\Phi}_k \neq 0, \quad (\widetilde{\Phi}_k)_{01} = v, \quad (\widetilde{\Phi}_k)_{02} = v', \tag{18}$$

with $a' = 1, \ldots, 35$.

Eq. (18) gives 22 free NGBs in total. Finally one has 8 scalar bosons which create 4 scalar doublets to be assigned as the SU(6) would-be Baby (Little) Higgs as follows [3],

$$\theta = \frac{1}{f}\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} (0)_{2\times2} & (h)_{2\times1} \\ (h'^{\dagger})_{1\times2} & 0 \end{pmatrix} \\ \begin{pmatrix} (0)_{2\times2} & (h')_{2\times1} \\ (h^{\dagger})_{1\times2} & 0 \end{pmatrix} & (0)_{3\times3} \end{pmatrix}, \tag{19}$$

where $f^2 = f_1^2 + f_2^2$. The scalar doublets h and h' are the would-be SM Higgs as will be explained later.

2.3 SU(6) would-be Baby (Little-like) Higgs

Let's start with an extra-dimensional scalar from the near-brane corresponding to 5D ($y \sim 0$) gauge boson within AdS/CFT correspondence of 4D global gauge and 5D local gauge symmetry [10] of which one takes Kaluza-Klein (KK) expansion with even and odd parities, one has the even fields [15],

$$\widetilde{\Phi}_+^{(i)}(x) = \frac{1}{\sqrt{\pi R}}\left[\widetilde{\Phi}_+^0(x)\right]_i + \sqrt{\frac{1}{\pi R}}\sum_{n=2}^{\infty}\left[\widetilde{\Phi}_+^n(x)\right]_i \cos\left(\frac{ny}{R}\right), \tag{20}$$

and the odd fields,

$$\widetilde{\Phi}_-^{(i)}(x) = \sqrt{\frac{1}{\pi R}}\sum_{n=1}^{\infty}\left[\widetilde{\Phi}_-^n(x)\right]_i \sin\left(\frac{ny}{R}\right), \tag{21}$$

where $\widetilde{\Phi}_{\pm}^{(i)}, i = 1, 2$ are the original bulk scalars.

At the orbifold singular points which are 4D and have the twist factor $T_g = +1$ for $y = 0$ and $T_g = -1$ for $y = \pi R$, one can assign, without loss of generality, the near-brane at $y \sim 0$. Defining further $i = 1$ and 2 for positive and negative exponential power one can establish $\widetilde{\Phi}^{(j)}, j = 1, 2$ with $(+)$ and $(-)$ subscripts respectively and obtain the wavefunction by setting $y \sim 0$ for $\cos\left(\frac{ny}{R}\right)$ as follows,

$$\begin{aligned}
\widetilde{\Phi}_+^{(1)}(x) = \widetilde{\Phi}_+^{(1)}(x,y)\Big|_{y\sim0} &= \frac{1}{\sqrt{\pi R}}\left\{\widetilde{\Phi}_{+,+}^{(0)}(x) + \widetilde{\Phi}_{+,+}^{(n)}(x)\right\}\cos\left(\frac{ny}{R}\right)\Big|_{y\sim0} \\
&= \frac{1}{\sqrt{\pi R}}\left[1 + \frac{if_2}{ff_1}\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h'^{\dagger} & & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & h' \\ 0 & 0 & \\ h^{\dagger} & & 0 \end{pmatrix} & (0)_{3\times3} \end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ f_1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \\
&\quad + \sqrt{\frac{1}{\pi R}}\sum_{n=2}^{\infty}\frac{1}{n!}\left[\frac{if_2}{ff_1}\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h'^{\dagger} & & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & h' \\ 0 & 0 & \\ h^{\dagger} & & 0 \end{pmatrix} & (0)_{3\times3} \end{pmatrix}\right]^n\begin{pmatrix} 0 \\ 0 \\ f_1 \\ 0 \\ 0 \\ 0 \end{pmatrix},
\end{aligned} \tag{22}$$

which has $e^{+i\alpha}$-form.

On the other hand, one finds that $\widetilde{\Phi}_{+}^{(2)}$ basically has $e^{-i\alpha}$-form, with the following expansion form as follows,

$$\begin{aligned}\widetilde{\Phi}_{+}^{(2)}(x)\Big|_{y\sim 0} = \widetilde{\Phi}_{+}^{(2)}(x,y) &= \frac{1}{\sqrt{\pi R}}\left\{\widetilde{\Phi}_{+,-}^{(0)}(x)+\widetilde{\Phi}_{+,-}^{(n)}(x)\right\}\cos\left(\frac{ny}{R}\right)\Big|_{y\sim 0}\\ &= \frac{1}{\sqrt{\pi R}}\left[1-\frac{if_1}{ff_2}\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0&0&h\\0&0&\\h'^{\dagger}&0& \end{pmatrix}\\ \begin{pmatrix} 0&0&h'\\0&0&\\h^{\dagger}&0& \end{pmatrix} & (0)_{3\times3}\end{pmatrix}\right]\begin{pmatrix}0\\0\\0\\0\\0\\f_2\end{pmatrix}\\ &+\sqrt{\frac{1}{\pi R}}\sum_{n=2}^{\infty}\frac{1}{(n)!}\left[-\frac{if_1}{ff_2}\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0&0&h\\0&0&\\h'^{\dagger}&0& \end{pmatrix}\\ \begin{pmatrix} 0&0&h'\\0&0&\\h^{\dagger}&0& \end{pmatrix} & (0)_{3\times3}\end{pmatrix}\right]^{n}\begin{pmatrix}0\\0\\0\\0\\0\\f_2\end{pmatrix}.\end{aligned} \tag{23}$$

The exponential forms of 5D ($y\sim 0$) near-brane or 4D SU(6) Little-like Higgses (scalars) $\widetilde{\Phi}_{+}^{(j)}$ follow directly from Eq. (22) and (23), and are named accordingly as SU(6) would-be Baby (Little) Higgses as follows,

$$\widetilde{\Phi}_{+}^{(1)}\left[\widetilde{\Phi}_{+}^{(2)}\right] = \frac{1}{\sqrt{\pi R}}e^{\frac{if_2}{ff_1}\left[\frac{-if_1}{ff_2}\right]\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0&0&h\\0&0&\\h'^{\dagger}&0& \end{pmatrix}\\ \begin{pmatrix} 0&0&h'\\0&0&\\h^{\dagger}&0& \end{pmatrix} & (0)_{3\times3}\end{pmatrix}}\begin{pmatrix}0\\0\\f_1\\0\\0\\0\end{pmatrix}\left[\begin{pmatrix}0\\0\\0\\0\\0\\f_2\end{pmatrix}\right]. \tag{24}$$

It clearly shows the similarity to the principle of PNBs parametrization in the Simplest Little Higgs [11,30,31]. The odd 5D gauge boson in Eq. (21) vanishes in ($y\sim 0$) near-brane due to $\sin\left(\frac{ny}{R}\right)=0$ for $y\sim 0$.

Next we are considering the wavefunction of SU(6) Little-like Higgs utilising the formula from non-linear sigma model and setting accordingly as,

$$\widetilde{\Phi}^{(1)}[\widetilde{\Phi}^{(2)}] = v[v']e^{\frac{if_2}{f_1}\left[-\frac{if_1}{f_2}\right]\theta}, \tag{25}$$

where $v[v']$ and θ are already defined in Eq (17) and (19).

One immediately retrieves back Eq. (24) from Eq. (25) after substituting $v[v']$ and θ. Unfortunately dimensional deconstruction without gauge symmetry breaking in 5D SU(6)→ 4D SU(6) which yields Eq. (22), (23) and (24) happens in 2 (two) equivalent manners i.e. trivial and pseudo non-trivial [13,14,26,30] where in the first no scalar is produced while the second produces scalar. This seemingly contradictory condition is what is exactly needed. The SU(6) would-be Baby (Little-like) Higgses can not exist too long as required by trivial manner from Eq. (4) righthand where $Z_2=U=I$, so that KK higher modes are eliminated naturally from Eq. (22), (23) leaving its zero modes as the allowed new scalar by pseudo non-trivial manner in Eq. (4) lefthand [3].

The new weakly-coupled scalars are, most likely, Higgs-like and the above approach by means of KK higher modes elimination is called weakly-approached.

This can be represented by SU(6) Baby Higgses which are defined by zero mode approximation in the lowest order perturbation. This scalar lives below energy scale $\Lambda_{(6)}^{\mathrm{NP}}$, SU(6) Baby Higgses can be written as (P : perturbative, NP: non-pertubative) [3],

$$\widetilde{\Phi}_{+,\mathrm{P}}^{(1)}(x) = v\Big(1+\frac{if_2}{f_1}\theta(x)\Big), \quad \widetilde{\Phi}_{+,\mathrm{P}}^{(2)}(x) = v'\Big(1-\frac{if_1}{f_2}\theta(x)\Big). \tag{26}$$

Eq. (26) brings us immediately to the orbifold-based field redefinition as follows,

$$\widetilde{\Phi}_{+,\mathrm{P}}^{(1)'}(x) = \widetilde{\Phi}_{+,\mathrm{P}}^{(1)}(x)-v+v', \quad \widetilde{\Phi}_{+,\mathrm{P}}^{(2)'}(x) = \widetilde{\Phi}_{+,\mathrm{P}}^{(2)}(x)-v'+v. \tag{27}$$

The new SU(6) Baby Higgses are surprisingly split into triplets of SU(3) Little-like Higgses in accordance to [3],

$$\widetilde{\Phi}^{(1)'}_{+,\mathrm{P}}(x) = \begin{pmatrix} 0_{3\times1} \\ \phi^{(1)}_{P} \end{pmatrix}, \quad \widetilde{\Phi}^{(2)'}_{+,\mathrm{P}}(x) = \begin{pmatrix} \phi^{(2)}_{P} \\ 0_{3\times1} \end{pmatrix}, \tag{28}$$

where SU(3) Little-like Higgses triplets are defined as

$$\phi^{(1)}_{\mathrm{P}} = \frac{1}{\sqrt{\pi R}} \left[\left\{ \left(1 + \frac{\Delta f}{f_1}\right) + \frac{i f_2}{f_1 f} \begin{pmatrix} 0 & 0 & h' \\ 0 & 0 & \\ & h^\dagger & 0 \end{pmatrix} \right\} \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix} \right], \tag{29}$$

$$\phi^{(2)}_{\mathrm{P}} = \frac{1}{\sqrt{\pi R}} \left[\left\{ \left(1 - \frac{\Delta f}{f_2}\right) - \frac{i f_1}{f_2 f} \begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ & h'^\dagger & 0 \end{pmatrix} \right\} \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix} \right], \tag{30}$$

where $\Delta f = f_2 - f_1$.

The potential of SU(6) Baby Higgses follows from Eq. (13) by replacing $\lambda^{(6)}_y \to \lambda^{(6)}_{\mu\mathrm{P}}$, with $i, j = 1, 2$, and $\widetilde{\Phi}^{(i)'}_{+,\mathrm{P}} \sim \widetilde{\Phi}^{(j)'}_{+,\mathrm{P}}$. From Eq. (28) one finds $\widetilde{\Phi}^{(i)'\dagger}_{+,\mathrm{P}} \widetilde{\Phi}^{(j)'}_{+,\mathrm{P}} = 0$ for $i \neq j$. Therefore Eq. (13) is rewritten as [3]

$$V^{(6)}_{\mu\mathrm{P}} = \delta_{ij} \lambda^{(6)}_{\mu\mathrm{P}} \widetilde{\Phi}^{(i)'\dagger}_{+,\mathrm{P}} \widetilde{\Phi}^{(j)'}_{+,\mathrm{P}} \widetilde{\Phi}^{(j)'\dagger}_{+,\mathrm{P}} \widetilde{\Phi}^{(i)'}_{+,\mathrm{P}}, \tag{31}$$

with $i, j = 1, 2$, and $\widetilde{\Phi}^{(i)'}_{+,\mathrm{P}} \sim \widetilde{\Phi}^{(j)'}_{+,\mathrm{P}}$ for $i \neq j$. From Eq. (28) one finds $\widetilde{\Phi}^{(i)'\dagger}_{+,\mathrm{P}} \widetilde{\Phi}^{(j)'}_{+,\mathrm{P}} = 0$ for $i \neq j$. Therefore Eq. (31) is rewritten as

$$V^{(6)}_{\mu\mathrm{P}} = \lambda^{(6)}_{\mu\mathrm{P}} \left\{ \left(\widetilde{\Phi}^{(1)'\dagger}_{+,\mathrm{P}} \widetilde{\Phi}^{(1)'}_{+,\mathrm{P}} \right)^2 + \left(\widetilde{\Phi}^{(2)'\dagger}_{+,\mathrm{P}} \widetilde{\Phi}^{(2)'}_{+,\mathrm{P}} \right)^2 \right\}, \tag{32}$$

and one finally arrives at the sum of potential of three Higgses. The mass -squareds and further discussion have been given in [3].

3 On the Higgs-like and Gauge-like scalars from generalized Coleman-Weinberg potential in SU(6) Near-brane

3.1 The strongly-coupled SU(6) will-be-SimplestLittle-Higgs Scalar

Let us start with the upper-near-brane Lagrangian which is basically a quartic potential [3] in Eq. (13) where $\lambda^{(6)}_y$ is coupling constant of the upper-near-brane while $\widetilde{\Phi}^{(i)}_+$ and $\widetilde{\Phi}^{(j)}_+$ are the SU(6)-origin would be Little (Baby) Higgs boson expressed as in Eq. (22) and Eq. (23) [3].

Lagrangian in the lower-near-brane can be obtained from Eq. (13), replacing $\lambda^{(6)}_y \to \lambda^{(6)}_{y\mathrm{NP}}$ and $\widetilde{\Phi}^{(i)}_+ \to \widetilde{\Phi}^{(i)'}_+, \widetilde{\Phi}^{(j)} \to \widetilde{\Phi}^{(j)'}$, $i = 1, 2$ and $j = 1, 2$, as follows,

$$\mathcal{L}^{\text{near-brane}}_y = V^{(6)}_{y\mathrm{NP}} = \lambda^{(6)}_{y\mathrm{NP}} \widetilde{\Phi}^{(i)'\dagger}_+ \widetilde{\Phi}^{(j)'} \widetilde{\Phi}^{(j)'\dagger} \widetilde{\Phi}^{(i)'}_+, \tag{33}$$

where $\lambda^{(6)}_{y\mathrm{NP}}$ is coupling constant of lower-near-brane. Now, one utilizes PNB Higgs representation in the exponential form as shown by Eq. (24) and (25) after dropping (+) index.

The $\widetilde{\Phi}^{(i)\prime}, \widetilde{\Phi}^{(j)\prime}$ are the SU(6) will-be-SimplestLittleHiggs bosons which can be expressed as [3],

$$\widetilde{\Phi}_+^{(1)\prime} = \widetilde{\Phi}^{(1)\prime} = \begin{pmatrix} \phi_0^{(1)} \\ \phi^{(1)} \end{pmatrix}, \quad \widetilde{\Phi}_+^{(2)\prime} = \widetilde{\Phi}^{(2)\prime} = \begin{pmatrix} \phi^{(2)} \\ \phi_0^{(2)} \end{pmatrix}, \tag{34}$$

and obtained, after applying another approach by means of expanding the (matrix) exponent into an (exponent) matrix, named accordingly as strongly-approached manner, as follows

$$e^{\frac{if_2}{f_1 f}\left[-\frac{if_1}{f_2 f}\right]\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h'^{\dagger} & 0 & \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & h' \\ 0 & 0 & \\ h^{\dagger} & 0 & \end{pmatrix} & (0)_{3\times3} \end{pmatrix}} \sim \begin{pmatrix} (1)_{3\times3} & e^{\frac{if_2}{f_1 f}\left[-\frac{if_1}{f_2 f}\right]\begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h'^{\dagger} & 0 & \end{pmatrix}} \\ e^{\frac{if_2}{f_1 f}\left[-\frac{if_1}{f_2 f}\right]\begin{pmatrix} 0 & 0 & h' \\ 0 & 0 & \\ h^{\dagger} & 0 & \end{pmatrix}} & (1)_{3\times3}, \end{pmatrix} \tag{35}$$

and multiplying with VEV sextets, surprisingly the SU(6) will-be-SimplestLittleHiggs scalars experience a splitting into weakly-coupled zero mode VEV triplet $\phi_0^{(1)}[\phi_0^{(2)}]$ and a strongly-coupled Little-like Higgs triplet $\phi^{(1)}[\phi^{(2)}]$ similar to triplet-doublet splitting of 5D SU(5) Higgs quintet [50].

One finds basically SU(3) VEV triplets for the weakly-coupled zero mode triplets and SU(3) Simplest-like Little Higgs for the strongly-coupled all-mode triplets with the following SU(3) VEV triplets,

$$\phi_0^{(1)}\left[\phi_0^{(2)}\right] = \frac{1}{\sqrt{\pi R}}\begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix}\left[\begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix}\right] = \begin{pmatrix} 0 \\ 0 \\ f_1' \end{pmatrix}\left[\begin{pmatrix} 0 \\ 0 \\ f_2' \end{pmatrix}\right], \tag{36}$$

and SU(3) Simplest-like Little Higgs with $\theta \to \theta'$ as

$$\phi^{(1)} = e^{\frac{if_2'}{f_1' f'}\begin{pmatrix} 0 & 0 & H' \\ 0 & 0 & \\ H^{\dagger} & 0 & \end{pmatrix}}\begin{pmatrix} 0 \\ 0 \\ f_1' \end{pmatrix}, \quad \phi^{(2)} = e^{-\frac{if_1'}{f_2' f'}\begin{pmatrix} 0 & 0 & H \\ 0 & 0 & \\ H'^{\dagger} & 0 & \end{pmatrix}}\begin{pmatrix} 0 \\ 0 \\ f_2' \end{pmatrix}, \tag{37}$$

where $f_i' = \frac{1}{\sqrt{\pi R}} f_i, f'^2 = f_1'^2 + f_2'^2, H(H') = \frac{1}{\sqrt{\pi R}} h(h')$, and

$$\theta' = \frac{1}{f'}\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0 & 0 & H \\ 0 & 0 & \\ H'^{\dagger} & 0 & \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & H' \\ 0 & 0 & \\ H^{\dagger} & 0 & \end{pmatrix} & (0)_{3\times3} \end{pmatrix}. \tag{38}$$

With these new variables Eq. (25) can be re-written as,

$$\widetilde{\Phi}^{(1)}[\widetilde{\Phi}^{(2)}] = v[v']e^{\frac{if_2'}{f_1'}\left[-\frac{if_1'}{f_2'}\right]\theta'}, \tag{39}$$

which can be expanded following Eq. (35), afterwards, one retrieves again Eq. (34) meaning that SU(6) will-be-SimplestLittleHiggs is another strongly-coupled Little-like Higgs suffering from large possibility to undergo triplet-triplet splitting.

One notices easily that SU(6) will-be-SimplestLittleHiggs scalars are just another forms of SU(6) would-be Baby (Little-like) Higgs i.e. the strongly-approached form. Therefore, under the requirements of trivial and pseudo non-trivial manners, $\widetilde{\Phi}^{(1)\prime}$ and $\widetilde{\Phi}^{(2)\prime}$ can not last too long and have to be converted into a newly-formed scalars which, in this strongly-approached manner, are achieved by the triplet-triplet splitting as shown above by Eq. (36) and (37) via strongly-approached expansion in Eq. (35).

In the next discussion the effect of triplet-triplet splitting and role of Simplet-like Little Higgs in building the potential are studied and explained of which the emerging of double vacua system is clearly indicated. On the other hand 2 (two) breaking patterns also show up as the result of different time of breakings of either shift symmetry or asymptotic shift symmetry, and also both (asymptotic) shift symmetry. The first with broken shift symmetry is named one-by-one breaking while the second consisting of the breakings of either only asymptotic shift symmetry or both (asymptotic and) shift symmetry are called collective breaking.

The one-by-one and collective patterns emerging from (asymptotic) shift symmetry can be seen clearly from the explanation of global gauge transformation at $\widetilde{\Phi}^{(i)}, i = 1,2$ by means of $e^{i\alpha Q}$ where Q is the broken generators. The (asymptotic) shift symmetry and its breaking terms can be seen from expressions, expanding $e^{i\alpha Q}\widetilde{\Phi}^{(i)}, i = 1,2$ with $\widetilde{\Phi}^{(i)}$ as given by Eq. (39), as below

$$\begin{aligned}\widetilde{\Phi}^{(1)} : \frac{f_2'}{f_1'}\theta' \rightarrow \frac{f_2'}{f_1'}\theta' + Q\alpha + i\left\{\frac{f_2'}{f_1'}\theta' Q\alpha + \big([1] + iQ\alpha\big)\frac{f_2'^2}{2f_1'^2}\theta'^2\right\} \\ + \big([1] + iQ\alpha\big)\mathcal{O}(\theta'^3),\end{aligned} \tag{40}$$

$$\begin{aligned}\widetilde{\Phi}^{(2)} : -\frac{f_1'}{f_2'}\theta' \rightarrow -\frac{f_1'}{f_2'}\theta' + Q\alpha - i\left\{\frac{f_1'}{f_2'}\theta' Q\alpha - \big([1] + iQ\alpha\big)\frac{f_1'^2}{2f_2'^2}\theta'^2\right\} \\ + \big([1] + iQ\alpha\big)\mathcal{O}(\theta'^3).\end{aligned} \tag{41}$$

the third term is the shift symmetry breaking term while the fourth term, or the (3rd and 4th together) is the asymptotic shift symmmetry breaking term.

3.2 Higgs-NGB double vacua system as a source of Higgs and gauge-like scalar

The AdS/CFT correspondence dictates the existing correspondence of 5D ($y \sim 0$) local gauge and 4D global gauge SU(6) symmetry [10] which, in this context, also conceives the near-brane uncertainty correspondence as depicted by Eq. (16). This means that the correspondence extends between 4D PNB Higgs and 5D ($y \sim 0$) Nambu-Goldstone boson (NGB) due to global and local properties. Consequently can one generalize and interprete $\widetilde{\Phi}^{(j)}, j = 1,2$ in Eq. (13) to include NGB, ξ, which can be put in a diagonal of matrix representation, taking consideration for PNBs residing in the off-diagonal of sub-matrices as shown by Eq. (19).

Therefore one can utilize, without loss of generality, SU(6) generators λ_8, λ_{34}, and λ_{35} for this purpose and defining NGBs as follows,

$$\xi^{(1)} = \xi(n_8\lambda_8 + n_{35}\lambda_{35}), \quad \xi^{(2)} = \xi(n_{34}\lambda_{34} + n_{35}\lambda_{35}) \tag{42}$$

where n_8, n_{34} and n_{35} are normalization constants and the generators are provided below

[1],

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} & (0)_{3\times 3} \\ (0)_{3\times 3} & \begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 \end{pmatrix} \end{pmatrix}, \ \lambda_{34} = \frac{1}{\sqrt{3}} \begin{pmatrix} \begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 \end{pmatrix} & (0)_{3\times 3} \\ (0)_{3\times 3} & \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \end{pmatrix},$$

$$\lambda_{35} = \frac{1}{\sqrt{3}} \begin{pmatrix} \begin{pmatrix} -1 & & \\ & -1 & \\ & & -1 \end{pmatrix} & (0)_{3\times 3} \\ (0)_{3\times 3} & \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \end{pmatrix}, \tag{43}$$

so that one obtains finally,

$$\xi^{(1)} = \xi \begin{pmatrix} 3 & & & & & \\ & 3 & & & & \\ & & 0 & & & \\ & & & -2 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix}, \quad \xi^{(2)} = \xi \begin{pmatrix} -2 & & & & & \\ & -2 & & & & \\ & & -2 & & & \\ & & & 3 & & \\ & & & & 3 & \\ & & & & & 0 \end{pmatrix}, \tag{44}$$

where ξ is NGB, making use definitions as follows: $\xi' = 3\xi, \xi_0 = -2\xi$ and $(\xi')_{\rm ng} = \begin{pmatrix} \xi' & & \\ & \xi' & \\ & & 0 \end{pmatrix}$, $\xi_0' = \xi_0 \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$, SU(6) NGBs in short form as,

$$\xi^{(1)} = \begin{pmatrix} ((\xi')_{\rm ng})_{3\times 3} & (0)_{3\times 3} \\ (0)_{3\times 3} & ((\xi')_0)_{3\times 3} \end{pmatrix}, \quad \xi^{(2)} = \begin{pmatrix} ((\xi')_0)_{3\times 3} & (0)_{3\times 3} \\ (0)_{3\times 3} & ((\xi')_{\rm ng})_{3\times 3}. \end{pmatrix} \tag{45}$$

Recalling Eq. (13) with $i \neq j$ Coleman-Weinberg potential is generalized by replacing $\widetilde{\Phi}^{(j)}, j = 1,2$ with $\Phi^{(j)}_{\rm NG}, j = 1,2$ which is quite justified considering $\Phi^{(j)}_{\rm NG}, j = 1,2$ is the third component of a massive 4D gauge boson while $\widetilde{\Phi}^{(i)}, i = 1,2$ PNB Higgs.

Substitution of the originally-set fifth component of 5D gauge boson with its third component is naturally logical and definitely acceptable [26] which gives a generalized Coleman-Weinberg potential for SU(6) would-be Baby (Little-like) Higgs as below,

$$V^{(6)}_{yg} = \lambda^{(6)}_y \big(\widetilde{\Phi}^{(i)\dagger}\Phi^{(j)}_{\rm NG}\big)\big(\Phi^{(j)\dagger}_{\rm NG}\widetilde{\Phi}^{(i)}\big), \quad i,j = 1,2, \tag{46}$$

and for SU(6) will-be-SimplestLittleHiggs as,

$$V^{(6)}_{{\rm NP}g} = \lambda^{(6)}_{y{\rm NP}} \big(\widetilde{\Phi}^{(i)'\dagger}\Phi^{(j)}_{\rm NG}\big)\big(\Phi^{(j)\dagger}_{\rm NG}\widetilde{\Phi}^{(i)'}\big), \quad i,j = 1,2, \tag{47}$$

where $\Phi^{(j)}_{\rm NG}$ is written as,

$$\Phi^{(j)}_{\rm NG} = v^{(j)}_{\rm NG} e^{\pm \frac{i f_i'}{f_j' f'} \xi^{(j)}}, \quad i \neq j = 1,2, \tag{48}$$

where $(+)$ for $i = 1, (-)$ for $i = 2$ and $v^{(1)}_{\rm NG} = v$ and $v^{(2)}_{\rm NG} = v'$ in Eq. (17) or $v^{(1)}_{\rm NG} = \begin{pmatrix} 0 & 0 & f_1' & 0 & 0 & 0 \end{pmatrix}^T$ and $v^{(2)}_{\rm NG} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & f_2' \end{pmatrix}^T$. Following Eq. (35) and expanding the (matrix) exponent into an (exponent) matrix in the strongly-approached manner one gets the result as,

$$
\begin{aligned}
e^{\pm\frac{if'_i}{f'_j f'}\xi^{(j)}} &= e^{-\frac{if'_2}{f'_1 f'}\left[\frac{if'_1}{f'_2 f'}\right]\begin{pmatrix}\xi'_{\rm ng} & 0\\ 0 & \xi'_0\end{pmatrix}\left[\begin{pmatrix}\xi'_0 & 0\\ 0 & \xi'_{\rm ng}\end{pmatrix}\right]} \\
&\sim \begin{pmatrix} e^{-\frac{if'_2}{f'_1 f'}\xi'_{\rm ng}} & 1 \\ 1 & e^{-\frac{if'_2}{f'_1 f'}\xi'_0}\end{pmatrix}\left[\begin{pmatrix} e^{\frac{if'_1}{f'_2 f'}\xi'_0} & 1 \\ 1 & e^{\frac{if'_1}{f'_2 f'}\xi'_{\rm ng}}\end{pmatrix}\right],
\end{aligned}
\tag{49}
$$

which brings directly to, with the aid of Eq. (17), the following sextets,

$$
\Phi^{(1)}_{\rm NG} = \begin{pmatrix} e^{-\frac{if'_2}{f'_1 f'}\xi'_{\rm ng}}\begin{pmatrix}0\\0\\f'_1\end{pmatrix} \\ \begin{pmatrix}0\\0\\f'_1\end{pmatrix}\end{pmatrix}, \quad \Phi^{(2)}_{\rm NG} = \begin{pmatrix} \begin{pmatrix}0\\0\\f'_2\end{pmatrix} \\ e^{\frac{if'_1}{f'_2 f'}\xi'_{\rm ng}}\begin{pmatrix}0\\0\\f'_2\end{pmatrix}\end{pmatrix}, \tag{50}
$$

where one can define SU(3) NGBs $\phi^{(j)}_{\rm NG}, j = 1, 2$ as follows,

$$
\phi^{(1)}_{\rm NG}[\phi^{(2)}_{\rm NG}] = e^{-\frac{if'_2}{f'_1 f'}\left[\frac{if'_1}{f'_2 f'}\right]\xi'_{\rm ng}}\begin{pmatrix}0\\0\\f'_1\end{pmatrix}\left[\begin{pmatrix}0\\0\\f'_2\end{pmatrix}\right], \quad \xi'_{\rm ng} = \begin{pmatrix}\xi' & & \\ & \xi' & \\ & & 0\end{pmatrix}. \tag{51}
$$

If factor $1/\sqrt{2}$ is allowed to be absorbed by $(\xi'_{\rm ng})_{jj} = \left(\frac{1}{\sqrt{2}}\xi'\right), j = 1, 2$, then Eq. (51) resembles completely NGBs produced in SU(3)×SU(3) symmetry as discussed in ref.[5].

As a matter of fact, one can rewrite Eq. (50) into a nice compact form similar to Eq. (34) as below,

$$
\Phi^{(1)}_{\rm NG} = \begin{pmatrix}\phi^{(1)}_{\rm NG}\\ \phi^{(1)}_0\end{pmatrix}, \quad \Phi^{(2)}_{\rm NG} = \begin{pmatrix}\phi^{(2)}_0\\ \phi^{(2)}_{\rm NG}\end{pmatrix} \tag{52}
$$

In comparison with Eq. (34) one finds that doublets in Eq. (52) above are the inversion of doublets in Eq. (34) after replacing $\phi^{(1)}_{\rm NG}$ with $\phi^{(1)}$, indicating 2 (two) different 2-VEV vacuum states where one belongs to PNB Higgs field and another one to NGB field or its derivates with 2-VEV doublets of the second is the inversion of the first's.

Summing up Eq. (34) and (52), the following result is obtained,

$$
\widetilde{\Phi}^{(1)\prime} + \Phi^{(1)}_{\rm NG} = \begin{pmatrix}\phi^{(1)}_0 + \phi^{(1)}_{\rm NG}\\ \phi^{(1)}_0 + \phi^{(1)}\end{pmatrix}, \quad \widetilde{\Phi}^{(2)\prime} + \Phi^{(2)}_{\rm NG} = \begin{pmatrix}\phi^{(2)}_0 + \phi^{(2)}\\ \phi^{(2)}_0 + \phi^{(2)}_{\rm NG}\end{pmatrix}, \tag{53}
$$

where each VEV $\phi^{(i)}_0, i = 1, 2$ is shifted by both Higgs shift and NGB shift indicating the existence of double vacua with 2-VEV system and inverted vacuum states [5,39].

3.3 Heisenberg scalar from generalized Coleman-Weinberg potential

With the aid of Eq. (39) and (48) the generalized Coleman-Weinberg potential in Eq. (13) is computed, taking $i = j = 1, 2$ for this purpose, so that one starts with,

$$
\begin{aligned}
V^{(6)} &= V^{(6)}_1 + V^{(6)}_2 \\
&= \lambda^{(6)}_y\left(\widetilde{\Phi}^{(1)\dagger}\Phi^{(1)}_{\rm NG}\right)\left(\Phi^{(1)\dagger}_{\rm NG}\widetilde{\Phi}^{(1)}\right) + \lambda^{(6)}_y\left(\widetilde{\Phi}^{(2)\dagger}\Phi^{(2)}_{\rm NG}\right)\left(\Phi^{(2)\dagger}_{\rm NG}\widetilde{\Phi}^{(2)}\right)
\end{aligned}
\tag{54}
$$

where $V^{(6)}_1$ and $V^{(6)}_2$ are found as below,

$$
V^{(6)}_1[V^{(6)}_2] = \lambda^{(6)}_y (v^T v)^2[(v'^T v')^2] e^{\frac{if'_2}{f'_1}\left[-\frac{if'_1}{f'_2}\right]\left\{(\theta' - \theta'^\dagger) + \frac{1}{f'}(\xi^{(1)\dagger}[\xi^{(2)}] - \xi^{(1)}[\xi^{(2)\dagger}])\right\}} \tag{55}
$$

with $\theta' = \theta'^\dagger$ and $\xi^{(i)} = \xi^{(i)\dagger}, i = 1, 2$ for SU(6). Special note must be taken here that scalars $\widetilde{\Phi}^{(i)\dagger}, \Phi^{(i)}, i = 1, 2$ in Eq. (54) are SU(6) would-be Baby (Little-like) Higgses which must decouple from the system. No wonder, the potentials in Eq. (55) are zero (or constant).

Only after triplet-triplet splitting which is induced by the emerging of SU(6) will-be-SimplestLittleHiggs scalars then potentials in Eq. (54) and (55) become non-zero which will be discussed in the next subsection. Meanwhile, it is adequate for the moment to consider the inequality between Eq. (13) and (33) is due to different physical conditions inherited from the changing from sextet-based to triplet-based as dictated by triplet-triplet splitting which provides finally the zero and non-zero potentials.

Consequently Eq. (55) must be adjustable and can be reformed to accommodate the non-zero potential, at least, at the switching point from Eq. (13) into (55). Let's rewrite Eq. (33) and (55) in the following compact form, setting $\lambda_y^{(6)} \sim \lambda_{y\mathrm{NP}}^{(6)}$, as

$$V_1^{(6)}[V_2^{(6)}] = \lambda_{y\mathrm{NP}}^{(6)} f_1'^4 [f_2'^4] e^{\frac{if_2'}{f_1'}\left[\frac{if_1'}{f_2'}\right]\left\{(\theta' \pm \frac{1}{f'}\xi^{(1)}[\xi^{(2)}]) - (\theta'^\dagger \pm \frac{1}{f'}\xi^{(1)\dagger}[\xi^{(2)\dagger}])\right\}} \tag{56}$$

where $\pm$ means $-\xi^{(1)}(\xi^{(1)\dagger})$ and $+\xi^{(2)}(\xi^{(2)\dagger})$ which are important property showing up in triplet-based due to $\theta' \neq \theta'^\dagger$ as shown below

$$\theta_1' = \frac{1}{f'}\begin{pmatrix} \begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix} & H' \\ H^\dagger & 0 \end{pmatrix}, \quad \theta_2' = \theta_1'^\dagger = \frac{1}{f'}\begin{pmatrix} \begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix} & H \\ H'^\dagger & 0 \end{pmatrix}, \tag{57}$$

while $\xi^{(1)}(\xi^{(2)})$ becomes $\xi_{ng}' = \xi_{ng}'^\dagger$. This is the reason to keep $\theta'^\dagger$ in Eq. (55) in order to have a more general form. The non-zero potential in Eq. (56) indicates (asymptotic) shift symmetry breaking and emerging of a new massive scalar which is defined accordingly as,

$$(H_H'') = (\theta_1' \pm \xi_{ng}') - (\theta_1'^\dagger \pm \xi_{ng}') \tag{58}$$

where (H_H'') is the will-be Heisenberg scalar in 3×3 matrix. Defining a new scalar, H'', which is a PNB Higgs (it will be clear in the next section) as follows,

$$H'' = H' - H, \quad H''^\dagger = H'^\dagger - H^\dagger \tag{59}$$

one can further define by rewriting Eq. (59) as [5],

$$H_H'' = (H' \pm \xi) - (H \pm \xi) \tag{60}$$

which shows the combination of Higgs and NGB, for the moment further discussion on this matter is pending until the next section, and is called as Heisenberg scalar. Comparing Eq. (58) and (60) one can establish a one-to-one correspondence as follows,

$$\theta'[\theta'^\dagger] \longleftrightarrow H'[H], \quad \xi_{ng}' \longleftrightarrow \xi, \quad (H_H'') \longleftrightarrow H_H'' \tag{61}$$

which shows convincingly that Heisenberg scalar is already hidden in the SU(6) global symmetry and emerges when asymptotic shift symmetry is broken as it will also be discussed later.

The basic property of Heisenberg scalar is clear from Eq. (60) where 2 (two) NGBs join and unify itself with a PNB Higgs $(H' - H)$. Therefore it has 2 (two) degrees of freedom in massless and 3 (three) in massive states. From this property one concludes that a Heisenberg scalar is basically a gauge-like scalar [5].

3.4 Masses of Heisenberg scalar and PNB Higgs from triplet-triplet splitting potential

With the aid of Eq. (33) and (54) the triplet-triplet splitting potential is obtained directly which is, in general, non-zero. This indicates the emerging of a massive particle which can be either a PNB Higgs or a Heisenberg scalar.

Let's start with rewriting the potential as,

$$V^{(6)}_{\mathrm{NP}g} = V^{(6)}_{\mathrm{NP}g(i=j)} + V^{(6)}_{\mathrm{NP}g(i\neq j)}, \tag{62}$$

$$V^{(6)}_{\mathrm{NP}g(i=j)} = \lambda^{(6)}_{y\mathrm{NP}} \left(\widetilde{\Phi}^{(i)\prime\dagger} \Phi^{(i)}_{\mathrm{NG}} \Phi^{(i)\dagger}_{\mathrm{NG}} \widetilde{\Phi}^{(i)\prime} + \Phi^{(i)\dagger}_{\mathrm{NG}} \widetilde{\Phi}^{(i)\prime} \widetilde{\Phi}^{(i)\prime\dagger} \Phi^{(i)}_{\mathrm{NG}} \right), \quad i,j = 1,2, \tag{63}$$

$$V^{(6)}_{\mathrm{NP}g(i\neq j)} = \lambda^{(6)}_{y\mathrm{NP}} \left(\widetilde{\Phi}^{(i)\prime\dagger} \Phi^{(j)}_{\mathrm{NG}} \Phi^{(j)\dagger}_{\mathrm{NG}} \widetilde{\Phi}^{(i)\prime} + \Phi^{(j)\dagger}_{\mathrm{NG}} \widetilde{\Phi}^{(i)\prime} \widetilde{\Phi}^{(i)\prime\dagger} \Phi^{(j)}_{\mathrm{NG}} \right), \quad i,j = 1,2. \tag{64}$$

Substituting Eq. (34) and (52) into Eq. (63), separating and neglecting the mixed-up terms such as $\phi^{(i)\dagger}_0 \phi^{(i)}_{\mathrm{NG}} \phi^{(i)\dagger}_0 \phi^{(i)}, \phi^{(i)\dagger} \phi^{(i)}_0 \phi^{(i)\dagger}_{\mathrm{NG}} \phi^{(i)}_0, \phi^{(i)\dagger}_{\mathrm{NG}} \phi^{(i)}_0 \phi^{(i)\dagger} \phi^{(i)}_0$ and $\phi^{(i)\dagger}_0 \phi^{(i)} \phi^{(i)\dagger}_0 \phi^{(i)}_{\mathrm{NG}}$ since it produces entities $(\theta' + \xi'_{\mathrm{ng}}), (\theta'^{\dagger} + \xi'_{\mathrm{ng}})$ and its linear combinations which facilitate double vacua with double shifts as shown by Eq. (53), and keep shift symmetry intact, one can finally simplify Eq. (63) becoming, replacing $\lambda^{(6)}_{y\mathrm{NP}}$ with $\lambda^{(3)}$, as

$$\begin{aligned} V^{(6)}_{\mathrm{NP}g(i=j)} = \lambda^{(3)} \Big\{ & (\phi^{(i)\dagger}_0 \phi^{(i)}_{\mathrm{NG}} \phi^{(i)\dagger}_{\mathrm{NG}} \phi^{(i)}_0 + \phi^{(i)\dagger}_{\mathrm{NG}} \phi^{(i)}_0 \phi^{(i)\dagger}_0 \phi^{(i)}_{\mathrm{NG}}) \\ & + (\phi^{(i)\dagger} \phi^{(i)}_0 \phi^{(i)\dagger}_0 \phi^{(i)} + \phi^{(i)\dagger}_0 \phi^{(i)} \phi^{(i)\dagger} \phi^{(i)}_0) \Big\} \end{aligned} \tag{65}$$

where $i = j = 1, 2$. The first term provides immediately the constant and the second term the mass term as follows,

$$V^{(6)}_{\mathrm{NP}g(i=j)} = 2(f_1'^4 + f_2'^4) + \frac{2 f_1'^2 f_2'^2}{f'^2} H''^{\dagger} H'', \tag{66}$$

where one finds convincingly that the mass term $H''^{\dagger} H''$ in Eq. (66) is yielded without any influence from NGB as clearly shown by no-NGB term in the second term. One names $H''_i, i = 1, 2$ as PNB Higgses with the mass-squared as follows,

$$m^2_{H''_i} = \frac{g'^4}{16\pi^2} \frac{\lambda^{(3)}}{f'^2} (2 f_1'^2 f_2'^2) \log \left(\frac{\Lambda^2_{(3)}}{\mu^2_{H''_i}} \right), i = 1, 2, \tag{67}$$

where $\mathcal{O}(\mu_{H''_i}) \sim \mathcal{O}(100\ \mathrm{GeV}), g'$ the SU(3) coupling constant, and $\Lambda_{(3)}$ cut-off scale.

On the contrary, one can also substitute Eq. (34) and (52) into Eq. (64) and find the mixed-up terms to be neglected, such as $\phi^{(i)\dagger}_0 \phi^{(j)}_0 \phi^{(j)\dagger}_{\mathrm{NG}} \phi^{(i)}, \phi^{(i)\dagger} \phi^{(j)}_{\mathrm{NG}} \phi^{(j)\dagger}_0 \phi^{(i)}_0, \phi^{(j)\dagger}_0 \phi^{(i)}_0 \phi^{(i)\dagger} \phi^{(j)}_{\mathrm{NG}}$ and $\phi^{(j)\dagger}_{\mathrm{NG}} \phi^{(i)} \phi^{(i)\dagger}_0 \phi^{(j)}_0$, based on the same reason as above, so that Eq. (64) is simplified, replacing $\lambda^{(6)}_{y\mathrm{NP}}$ with $\lambda^{(3)}$, as below,

$$\begin{aligned} V^{(6)}_{\mathrm{NP}g(i\neq j)} = \lambda^{(3)} \Big\{ & (\phi^{(i)\dagger}_0 \phi^{(j)}_0 \phi^{(j)\dagger}_0 \phi^{(i)}_0 + \phi^{(j)\dagger}_0 \phi^{(i)}_0 \phi^{(i)\dagger}_0 \phi^{(j)}_0) \\ & + (\phi^{(i)\dagger} \phi^{(j)}_{\mathrm{NG}} \phi^{(j)\dagger}_{\mathrm{NG}} \phi^{(i)} + \phi^{(j)\dagger}_{\mathrm{NG}} \phi^{(i)} \phi^{(i)\dagger} \phi^{(j)}_{\mathrm{NG}}) \Big\}, \end{aligned} \tag{68}$$

where $i \neq j = 1, 2$. The first term is just a constant while the second the anticipated mass term as shown below,

$$V^{(6)}_{\mathrm{NP}g(i\neq j)} = 4(f_1'^2 f_2'^2) + \frac{f_1'^4 + f_2'^4}{f'^2} H''^{\dagger} H'', \tag{69}$$

where one finds without any doubt that the mass term $H''^{\dagger}H''$ in Eq. (69) is obtained from the previously-cited source i.e. the combination of Higgs and NGBs as shown by the second term of Eq. (68), which is actually the previously-defined Heisenberg scalar, H''_H, with the following mass-squared,

$$m^2_{H''_H} = \frac{g'^4}{16\pi^2}\frac{\lambda^{(3)}}{f'^2}(f_1'^4 + f_2'^4)\log\left(\frac{\Lambda^2_{(3)}}{\mu^2_{H''_H}}\right), \tag{70}$$

where $\mathcal{O}(\mu_{H''_H}) \sim \mathcal{O}(100\ \text{GeV})$.

Both PNB Higgs and Heisenberg scalar become the basic constituent of exotic scalars where the formation of Heisenberg scalar from a Higgs and 2 (two) NGBs including the mass generation requires a new mechanism so-called Heisenberg Commutator-Uncertainty mechanism which will be discussed in a separate paper [5].

Before closing the subsection, one is reminded that Eq. (70) does show a stand-alone free Heisenberg scalar and can be regarded as a gauge-like single scalar which also, quite possibly, can serve as candidate of a relic dark matter [12,20]. Nevertheless one also faces the hidden Heisenberg scalar which can basically be found as a bonded scalar, jointed with either PNB Higgses or itselves to form exotic multi-component Higgses and scalars. This is to be discussed in the next section.

4 Hidden Heisenberg scalar and exotic 3-component scalar

4.1 The triplet-triplet splitting potential of exotic scalars

Near-brane Coleman-Weinberg potential shows special property as mentioned before with $i = j$ corresponding to Higgs-like scalar while $i \neq j$ to gauge-like scalar. This is due to uncertainty-based global-local gauge correspondence as shown by Eq. (16) , based on which, the potential can be grouped into $V^{(6)}_{y\text{NP}}(i = j)$ and $V^{(6)}_{y\text{NP}}(i \neq j)$. Therefore Eq. (33) can be rewritten in the following form,

$$V^{(6)}_{y\text{NP}} = V^{(6)}_{y\text{NP}}(i = j) + V^{(6)}_{y\text{NP}}(i \neq j), \tag{71}$$

$$V^{(6)}_{y\text{NP}}(i = j) = \lambda^{(6)}_{y\text{NP}}\left(\widetilde{\Phi}^{(i)'\dagger}_{+}\widetilde{\Phi}^{(i)'}\right)^2, \quad i = 1,2, \tag{72}$$

$$V^{(6)}_{y\text{NP}}(i \neq j) = \lambda^{(6)}_{y\text{NP}}\left(\widetilde{\Phi}^{(1)'\dagger}_{+}\widetilde{\Phi}^{(2)'}\widetilde{\Phi}^{(2)'\dagger}\widetilde{\Phi}^{(1)'}_{+} + \widetilde{\Phi}^{(2)'\dagger}_{+}\widetilde{\Phi}^{(1)'}\widetilde{\Phi}^{(1)'\dagger}\widetilde{\Phi}^{(2)'}_{+}\right), \tag{73}$$

where $\widetilde{\Phi}^{(i)'}_{+}, i = 1,2$ represents PNB Higgs while $\widetilde{\Phi}^{(j)'}, j = 2,1$ corresponds to exotic scalar. Therefore Eq. (72) shows straightforwardly the mass terms of Higgs-like scalars, indicating a single vacuum as a source of non-zero VEVs, as shown below,

$$V^{(6)}_{y\text{NP}(1)} = \lambda^{(6)}_{y\text{NP}(i=j=1)} = 2\big(\phi^{(1)\dagger}_0\phi^{(1)}_0\big)\phi^{(1)\dagger}\phi^{(1)} + \big(\phi^{(1)\dagger}\phi^{(1)}\big)^2, \tag{74}$$

$$V^{(6)}_{y\text{NP}(2)} = \lambda^{(6)}_{y\text{NP}(i=j=2)} = 2\phi^{(2)\dagger}\phi^{(2)}\big(\phi^{(2)\dagger}_0\phi^{(2)}_0\big) + \big(\phi^{(2)\dagger}\phi^{(2)}\big)^2. \tag{75}$$

Unfortunately there is no guarantee that potentials in both Eq. (74) and (75) exist at the same time eventhough coming from the same vacuum. To see this possibility one recalls Eq. (40) and (41) where shift symmetry and asymptotic shift symmetry breaking terms are shown in the expansions of $e^{iQ\alpha}\widetilde{\Phi}^{(1)'}$ and $e^{iQ\alpha}\widetilde{\Phi}^{(2)'}$.

In case the term $\big([1] + iQ_i\alpha\big)\frac{f_i'^2}{f_j'^2}\theta_i'^2, i \neq j = 1,2$ becomes significant with respect to $\frac{f_i'}{f_j'}\theta_i' Q_i\alpha, i \neq j = 1,2$ then $\widetilde{\Phi}^{(1)'}_{+}$ (with its element $\phi^{(1)}$) and $\widetilde{\Phi}^{(2)'}_{+}$ (with its element $\phi^{(2)}$) in

Eq. (40) and (41) have different rates in achieving the shift symmetry breaking. Hence, one has roughly a one-by-one breaking pattern.

Eq. (72) can also be expressed as follows

$$V^{(6)}_{y\mathrm{NP}(i=j)} = V^{(6)}_{y\mathrm{NP}(0)} + V^{(6)}_{y\mathrm{NP}(1)} + V^{(6)}_{y\mathrm{NP}(2)}, \tag{76}$$

where $V^{(6)}_{y\mathrm{NP}(0)} = f_1'^4 + f_2'^4$ which is neglectable at correct benchmarking.

Consequently we treat separately each remaining term, for farther-from-brane part, and after substituting Eq. (36) and Eq. (37) into Eq. (74), (75) and further into (76) one finds as follows,

$$V^{(6)}_{y\mathrm{NP}(1)}\left[V^{(6)}_{y\mathrm{NP}(2)}\right] = \lambda^{(6)}_{y\mathrm{NP}}\Big(\frac{2f_1'^2 f_2'^2}{f'^2} + \frac{f_2'^4[f_1'^4]}{4f'^4}(v'')^2\Big)H''^{\dagger}H'' + \lambda^{(6)}_{y\mathrm{NP}}\frac{f_2'^4[f_1'^4]}{4f'^4}(H''^{\dagger}H'')^2, \tag{77}$$

where we use the following definition, $H'' = H' - H$, for Higgs-like scalar,

$$\langle H''\rangle = \tfrac{1}{\sqrt{2}}\begin{pmatrix}0\\ v''\end{pmatrix}, \quad \mathrm{Tr}(H''H''^{\dagger}) = \tfrac{1}{2}(v'')^2, \tag{78}$$

with $\mathcal{O}(v'') \sim \mathcal{O}(100\,\mathrm{GeV})$.

With $m^2_{H_1''} \sim m^2_{H_2''}$ it is rewritten as $m^2_{H''_{i(\mathrm{obo})}}, i = 1, 2$ to give as follows,

$$m^2_{H''_{i(\mathrm{obo})}} = \frac{g'^4}{16\pi^2}\frac{\lambda^{(3)}}{f'^2}(2f_1'^2 f_2'^2)\log\left(\frac{\Lambda^2_{(3)}}{\mu^2_{H''_{i(\mathrm{obo})}}}\right), i = 1, 2, \tag{79}$$

$\mathcal{O}(\mu_{H''_{i(\mathrm{obo})}}) \sim \mathcal{O}(100\,\mathrm{GeV})$. This is exactly the PNB Higgs shown in Eq. (67) being reproduced in one-by-one breaking under potential $V^{(6)}_{y\mathrm{NP}}, i = j = 1, 2$. If one replaces SU(6) parameters: $\Lambda^{\mathrm{ZP}}_{(6)}, g$ and $\lambda^{6}_{\mu\mathrm{P}}$ with SU(3) parameters: $\Lambda_{(3)}, g'$ and $\lambda^{(3)}$ then Eq.(79) can be regarded as a pair of SU(6)-origin light Higgses brought from SU(6)-level [3] down to SU(3)-level. This pairing condition is important for understanding multi-component Higgs which shows up as an intermediate Higgs.

On the contrary, for collective breaking pattern, asymptotic and shift symmetry breakings happen for $\widetilde{\Phi}^{(1)'}$ and $\widetilde{\Phi}^{(2)'}$ at the same time requiring necessary condition $f_1' \sim f_2'$ and contribution of its elements, $\phi^{(1)}$ and $\phi^{(2)}$, takes place simultaneously. In this patterns $V^{(6)}_{y\mathrm{NP}}(i \neq j)$ can be rewritten in split triplets with $i \neq j = 1, 2$ and factors $\phi_0^{(i)}\phi^{(j)}, \phi^{(i)}\phi_0^{(j)}$ as follows,

$$\begin{aligned} V^{(6)}_{y\mathrm{NP}1(i\neq j)} &= \lambda^{(6)}_{y\mathrm{NP}}\big(\widetilde{\Phi}^{(1)'\dagger}_{+}\widetilde{\Phi}^{(2)'}\big)\big(\widetilde{\Phi}^{(2)'\dagger}\widetilde{\Phi}^{(1)'}_{+}\big) \\ &= \lambda^{(6)}_{y\mathrm{NP}}\Big\{(\phi_0^{(1)\dagger}\phi^{(2)})(\phi^{(2)\dagger}\phi_0^{(1)}) + (\phi^{(1)\dagger}\phi_0^{(2)})(\phi_0^{(2)\dagger}\phi^{(1)}) + \\ &\quad + (\phi_0^{(1)\dagger}\phi^{(2)})(\phi_0^{(2)\dagger}\phi^{(1)}) + (\phi^{(1)\dagger}\phi_0^{(2)})(\phi^{(2)\dagger}\phi_0^{(1)})\Big\} \end{aligned} \tag{80}$$

$$\begin{aligned} V^{(6)}_{y\mathrm{NP}2(i\neq j)} &= \lambda^{(6)}_{y\mathrm{NP}}\big(\widetilde{\Phi}^{(2)'\dagger}_{+}\widetilde{\Phi}^{(1)'}_{+}\big)\big(\widetilde{\Phi}^{(1)'\dagger}\widetilde{\Phi}^{(2)'}_{+}\big) \\ &= \lambda^{(6)}_{y\mathrm{NP}}\Big\{(\phi_0^{(2)\dagger}\phi^{(1)})(\phi^{(1)\dagger}\phi_0^{(2)}) + (\phi^{(2)\dagger}\phi_0^{(1)})(\phi_0^{(1)\dagger}\phi^{(2)}) + \\ &\quad + (\phi_0^{(2)\dagger}\phi^{(1)})(\phi_0^{(1)\dagger}\phi^{(2)}) + (\phi^{(2)\dagger}\phi_0^{(1)})(\phi^{(1)\dagger}\phi_0^{(2)})\Big\}. \end{aligned} \tag{81}$$

Adding and simplifying Eq. (80) and (81), only for second terms of the second equations, by defining the following,

$$V^{(3)}_{\mathrm{H}(1)} = \lambda^{(6)}_{y\mathrm{NP}}\Big\{(\phi_0^{(1)\dagger}\phi^{(2)})(\phi^{(2)\dagger}\phi_0^{(1)}) + (\phi_0^{(2)\dagger}\phi^{(1)})(\phi^{(1)\dagger}\phi_0^{(2)})\Big\}, \tag{82}$$

$$V^{(3)}_{\mathrm{H}(2)} = \lambda^{(6)}_{y\mathrm{NP}}\Big\{(\phi^{(1)\dagger}\phi_0^{(2)})(\phi_0^{(2)\dagger}\phi^{(1)}) + (\phi^{(2)\dagger}\phi_0^{(1)})(\phi_0^{(1)\dagger}\phi^{(2)})\Big\}, \tag{83}$$

which gives the mass terms and constant potentials, after adjusting $\lambda^{(6)}_{y\mathrm{NP}} \rightarrow \lambda^{(3)}$, as

$$V^{(3)}_{H''} = \lambda^{(3)} \left\{ (4f_1'^2 f_2'^2) + \frac{f_1'^4 + f_2'^4}{2f'^2} v''^2 + \frac{f_1'^4 + f_2'^4}{f'^2} H''^{\dagger} H'' \right\}, \tag{84}$$

where $\mathcal{O}(v'') \sim \mathcal{O}(100\ \mathrm{GeV})$ and $V^{(3)}_{H''} = V^{(3)}_{H(1)} + V^{(3)}_{H(2)}$. The remaining terms of Eq. (80) and (81) provide accordingly as follows,

$$V^{(3)}_{\mathrm{C}(1)} = \lambda^{(6)}_{y\mathrm{NP}} \left\{ (\phi_0^{(1)\dagger}\phi^{(2)})(\phi_0^{(2)\dagger}\phi^{(1)}) + (\phi_0^{(2)\dagger}\phi^{(1)})(\phi_0^{(1)\dagger}\phi^{(2)}) \right\}, \tag{85}$$

$$V^{(3)}_{\mathrm{C}(2)} = \lambda^{(6)}_{y\mathrm{NP}} \left\{ (\phi^{(1)\dagger}\phi_0^{(2)})(\phi^{(2)\dagger}\phi_0^{(1)}) + (\phi^{(2)\dagger}\phi_0^{(1)})(\phi^{(1)\dagger}\phi_0^{(2)}) \right\}, \tag{86}$$

which can be expressed, setting $\Delta f' = f_2' - f_1' (f_2' > f_1')$, $\frac{f_2'}{f_1'}\left[\frac{f_1'}{f_2'}\right] = 1 + \frac{1}{f_1'}\left[-\frac{1}{f_2'}\right]\Delta f'$ and $\lambda^{(6)}_{y\mathrm{NP}} \rightarrow \lambda^{(3)}$, as follows,

$$V^{(3)}_{\mathrm{C}(1)} = \lambda^{(3)}(2f_1'^2 f_2'^2)\, e^{\left[\frac{i}{f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^{\dagger} & 0 \end{pmatrix} + \frac{i\Delta f'}{f'}\left\{\frac{1}{f_1'}\begin{pmatrix} 0\,0 & H' \\ 0\,0 & \\ H^{\dagger} & 0 \end{pmatrix} + \frac{1}{f_2'}\begin{pmatrix} 0\,0 & H \\ 0\,0 & \\ H'^{\dagger} & 0 \end{pmatrix}\right\}\right]}, \tag{87}$$

$$V^{(3)}_{\mathrm{C}(2)} = \lambda^{(3)}(2f_1'^2 f_2'^2)\, e^{\left[\frac{i}{f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^{\dagger} & 0 \end{pmatrix} - \frac{i\Delta f'}{f'}\left\{\frac{1}{f_2'}\begin{pmatrix} 0\,0 & H' \\ 0\,0 & \\ H^{\dagger} & 0 \end{pmatrix} + \frac{1}{f_1'}\begin{pmatrix} 0\,0 & H \\ 0\,0 & \\ H'^{\dagger} & 0 \end{pmatrix}\right\}\right]}. \tag{88}$$

For collective breaking $\Delta f' \sim 0$ leaving only the first terms in exponents which provide mass terms, after expanding each exponent in Eq. (87), (88) and adding all terms up to second order into a single expression, as below,

$$V^{(3)}_{\mathrm{C}} = V^{(3)}_{\mathrm{C}(1)} + V^{(3)}_{\mathrm{C}(2)} = \lambda^{(3)} \left\{ 4f_1'^2 f_2'^2 + \frac{(f_1'^2 f_2'^2)}{f'^2} v''^2 + \frac{(2f_1'^2 f_2'^2)}{f'^2} H''^{\dagger} H'' \right\}. \tag{89}$$

Considering H'' as Higgs-like scalar which will be explained later one can finally rewrites Eq. (84) and (89) in mass term, neglecting constant potential, as follows,

$$V^{(3)}_{y\mathrm{NP}} = V^{(3)}_{\mathrm{C}} + V^{(3)}_{\mathrm{H''}} = \frac{\lambda^{(3)}}{f'^2} \left\{ 2f_1'^2 f_2'^2 + (f_1'^4 + f_2'^4) \right\} H''^{\dagger} H''. \tag{90}$$

One should have written the mass-coupling $\left\{ 2f_1'^2 f_2'^2 + (f_1'^4 + f_2'^4) \right\}$ as a quadrat $(f_1'^2 + f_2'^2)^2 = f'^4$, however, the reason to keep as it is will be clear soon if one notices the term $(2f_1'^2 f_2'^2)$ is the mass-coupling of one-by-one breaking, on the other side, the term $(f_1'^4 + f_2'^4)$ must belong to collective breaking, in other words, the mass-coupling of Eq. (90) shows clearly the double vacua property consisting of 2 (two) different components, as shown by the original Coleman-Weinberg potential $V^{(6)}_{i \neq j}$, where i corresponds to PNB Higgs and j to exotic scalar, so that (i, j) does not represent a single vacuum but rather the existence of double vacua.

4.2 The 3-component scalar and its basic constituent

Now, it's time to verify that the mass-coupling $\frac{\lambda^3}{f'^2}(f_1'^4 + f_2'^4)$ really belongs to a hidden Heisenberg scalar. Let's start with the breaking of global symmetry 4D SU(6)→SU(3)× SU(3)× U(1) in near-brane (or 5D with $y \sim 0$) which is accompanied by the local gauge symmetry breaking, that produces 18 broken generators with each corresponds to NGB. Each NGB pair can be connected directly to a massless PNB Higgs, H_1'' or H_2'', as shown in Eq. (60), each NGB is responsible for transfer of mass and 2 (two) degrees of freedom to each PNB Higgs. Now, a discussion of the role of the total 9 (nine) NGB pairs is given.

Without loss of generality, one may allocate 8 (eight) pairs of NGB for one SU(3) giving $\mathrm{SU(3)_{HS}}$ symmetry covering exotic scalars with Heisenberg scalar as its constituents and 1 (one) pair for U(1) since its breaking indicates the absence of one-by-one breaking, or no PNB Higgs, and the emerging of Heisenberg scalar as the replacement, and it is labeled as $\mathrm{U(1)_{TE}}$ (TE: time elapse). This gives, labeling $\mathrm{SU(3)_{PH}}$ for PNB Higgs symmetry, the new global symmetry shows up as $\mathrm{SU(3)_{HS}}\times\mathrm{SU(3)_{PH}}\times\mathrm{U(1)_{TE}}$ which will be discussed, together with a newly-developed Heisenberg Commutator-Uncertainty mechanism for generating exotic scalar masses, in a separate paper [5].

On the other side, the 8 (eight) pairs can also be distributed equally to both $\mathrm{SU(3)_s}$, each SU(3) can absorb 4 (four) pairs. This treatment regards the double vacua as having 2 (two) same types of vacua i.e. both are sources of exotic scalars in contrast to the first where each vacuum is totally separated and different.

The second one is to be discussed here, utilizing exponential forms of Eq. (82) and (83) for $V^{(3)}_{H(i)}, i = 1, 2$ and Eq. (87) and (88) for $V^{(3)}_{C(i)}, i = 1, 2$.

4.2.1 The 3-scalar Higgs

The potential in Eq. (90) provides directly the mass-squared of the exotic scalar constituted of 2 (two) basic constituents as guided by the previously-cited double vacua for PNB Higgs and exotic scalar. From the discussion in Section 3 especially 3.2. to 3.4 it is clearly indicated that the first is PNB Higgs while the second is Heisenberg scalar which are strongly supported by the same mass couplings as given in Eq. (67) and (70). One writes the mass-squared of the multi-component scalar directly from Eq. (90) as follows,

$$m^2_{H''_{col}} = \frac{g'^4}{16\pi^2}\frac{\lambda^{(3)}}{f'^2}\{2f_1'^2 f_2'^2 + (f_1'^4 + f_2'^4)\}\log\left(\frac{\Lambda^2_{(3)}}{\mu^2_{H''_{col}}}\right), \tag{91}$$

where $\mathcal{O}(\mu_{H''_{col}}) \sim \mathcal{O}(100\ \mathrm{GeV})$.

Let's continue with the allocation of 4 (four) pairs of NGB to one SU(3). With the aid of Eq. (68) one rewrites Eq. (82) and (83) by substituting $\phi_0^{(j)}[\phi_0^{(j)\dagger}] \to \phi_{\mathrm{NG}}^{(j)}[\phi_{\mathrm{NG}}^{(j)\dagger}]$ and finds that Eq. (82) and (83) are equivalent with the 2nd term of Eq. (68) which yields a massive Heisenberg scalar. One concludes here, beyond any doubt, that hidden Heisenberg scalars do exist and reside in both equations. Consequently, one substitutes $H'' \to H''_0$ to represent the hidden (or bonded) Heisenberg scalar in a multi-component scalar and rewrites as below,

$$V^{(3)}_{\mathrm{H}(2)} = \lambda^{(3)}(2f_1'^2 f_2'^2)e^{\frac{if_2'}{f_1'f'}\begin{pmatrix} \begin{smallmatrix}0&0\\0&0\end{smallmatrix} & H''_0 \\ -H''^{\dagger}_0 & 0\end{pmatrix}}, \tag{92}$$

$$V^{(3)}_{\mathrm{H}(1)} = \lambda^{(3)}(2f_1'^2 f_2'^2)e^{\frac{if_1'}{f_2'f'}\begin{pmatrix} \begin{smallmatrix}0&0\\0&0\end{smallmatrix} & H''_0 \\ -H''^{\dagger}_0 & 0\end{pmatrix}}. \tag{93}$$

where now,

$$H''_0[H''^{\dagger}_0] = (H' \pm \xi)[(H'^{\dagger} \pm \xi)] - (H \pm \xi)[(H^{\dagger} \pm \xi)]. \tag{94}$$

If one expands Eq. (92) and (93) to the second order, adds both equations, takes the mass terms and contants, to finally find again Eq. (84). Now, it is crystal clear that the second mass-coupling in Eq. (91) is really a Heisenberg scalar as shown in Eq. (70) with the only difference comes from being bonded instead of being free (stand-alone). Therefore H''_0 is named as bonded Heisenberg scalar having 2 (two) degrees of freedom in massless state and 3 (three) degrees of freedom in massive state, if it is bonded by 2 (two) PNB Higgses. Now, on the contrary, the other 4 (four) NGB pairs correspond to another SU(3) which exhibits a more general functions as the source of exotic scalars i.e. the hybrid source producing either a PNB Higgs scalar-pair or a Heisenberg scalar-pair. From Eq. (60) and (94) it is clear that every 2 (two) NGBs, each comes from $\phi_{\rm NG}^{(1)}[\phi_{\rm NG}^{(2)\dagger}]$, are absorbed by H and H' through H''_H and H''_0.

Unfortunately, Eq. (87) and (88) contain $H[H^\dagger], H'[H'^\dagger]$ and $H''[H''^\dagger]$ so that 4 (four) NGB pairs can be absorbed by either $H[H^\dagger]$ and $H'[H'^\dagger]$ in one group or $H''[H''^\dagger]$ in another group.

To further investigate one recalls global-local gauge near-brane Uncertainty Correspondence as depicted by Eq. (16), $\Delta\alpha(x) \sim \alpha$, from where a very small α-value corresponding to asymptotic shift symmetry may increase significantly ($\Delta\alpha \sim \alpha$), but still keeping α-value small enough while $\Delta\alpha(x)$, in comparison to $\alpha(x)$, increases comparably to α. This first condition means that asymptotic shift symmetry and local gauge symmetry are broken but global gauge symmetry which is triggered by shift symmetry remains intact [5]. The second case is dictated by a relatively large α-value where any α-value increase makes both $\Delta\alpha(x)$ and α itself becoming more significant and requires a collective (simultaneous) breaking of asymptotic shift-local gauge symmetry and global gauge symmetry breaking. This makes possible the NGB absorption directly by $H[H^\dagger]$ and $H'[H'^\dagger]$ under the condition $f'_1 \sim f'_2$ or $\left(\frac{1}{f'_1}\xi - \frac{1}{f'_2}\xi\right) \sim 0$. Eq. (87) and (88) become accordingly as below,

$$V_{\rm C(1)}^{(3)\rm col} = \lambda^{(3)}(2f_1'^2 f_2'^2) e^{\left[\frac{i}{f'}\begin{pmatrix} 0\,0 & H''_i \\ 0\,0 & \\ -H_i''^\dagger & 0 \end{pmatrix} + \frac{i\Delta f'}{f'}\left\{\frac{1}{f'_1}\begin{pmatrix} 0\,0 & (H'+\xi) \\ 0\,0 & \\ (H^\dagger+\xi) & 0 \end{pmatrix} + \frac{1}{f'_2}\begin{pmatrix} 0\,0 & (H-\xi) \\ 0\,0 & \\ (H'^\dagger-\xi) & 0 \end{pmatrix}\right\}\right]}, \tag{95}$$

$$V_{\rm C(2)}^{(3)\rm col} = \lambda^{(3)}(2f_1'^2 f_2'^2) e^{\left[\frac{i}{f'}\begin{pmatrix} 0\,0 & H''_i \\ 0\,0 & \\ -H_i''^\dagger & 0 \end{pmatrix} - \frac{i\Delta f'}{f'}\left\{\frac{1}{f'_2}\begin{pmatrix} 0\,0 & (H'+\xi) \\ 0\,0 & \\ (H^\dagger+\xi) & 0 \end{pmatrix} + \frac{1}{f'_1}\begin{pmatrix} 0\,0 & (H-\xi) \\ 0\,0 & \\ (H'^\dagger-\xi) & 0 \end{pmatrix}\right\}\right]}, \tag{96}$$

where $i = 1, 2$ for both Eq. (95) and (96).

Fortunately, under necessary condition of collective breaking $f'_1 \sim f'_2(\Delta f' \sim 0)$ the absorbed NGB and $H[H']$ vanish immediately and leave potential $V_{\rm C}^{(3)}$ in terms of PNB Higgs H''_i as,

$$V_{\rm C}^{(3)(\rm col)} = \lambda^{(3)}(4f_1'^2 f_2'^2) e^{\frac{i}{f'}\begin{pmatrix} 0\,0 & H''_i \\ 0\,0 & \\ -H_i''^\dagger & 0 \end{pmatrix}}, \quad i = 1, 2, \tag{97}$$

where $H''_i, i = 1, 2$ PNB Higgs just like in the Eq. (67) and also in the one-by-one breaking result in Eq. (79) as clearly shown by Eq. (97) with expansion to second order which is already given in Eq. (89) above.

Finally one can rewrite total potential $V_{y\rm NP(i\neq j)}^{(6)}$, making use the difference between H''_0 and $H''_i, i = 1, 2$ for Heisenberg scalar and PNB Higgs respectively, as

$$V_{y\rm NP(i\neq j)}^{(6)} = V_{H''}^{(3)}(H''_0) + V_C^{(3)(\rm col)}(H''_i), \quad i = 1, 2, \tag{98}$$

which explains the mass-squared in Eq. (91) to consist of PNB Higgs for $(2f_1'^2 f_2'^2)$ and Heisenberg scalar for $(f_1'^4 + f_2'^4)$. Of course, the mass-squared of H''_0 is obtained immediately from Eq. (90) or be found directly from Eq. (70) after substituting $H''_H \to H''_0$. Eq. (98) reminds us about the scalar-pair $H''_i, i = 1, 2$ where a pair of PNB Higgses lives and resides in a very compact space (not just a doublet) which impacts directly the condition of multi-component scalar in Eq. (91), now, it can be regarded as a 3-scalar Higgs, beyond any doubt, with one free degree of freedom. Schematic drawing is given in Appendix B showing 3-scalar Higgs like a heliumic atom and rather strongly-coupled scalar.

If $H \sim H'$ or $H_i'' = (H' - H)_i \sim 0, i = 1, 2$ potential $V_{\rm C}^{(3)}(H_i'')$ becomes constant and only $V_{H''}^{(3)}(H_0'')$ can produce in Eq. (98) substituting $H_0'' \to H_H''$, for a free Heisenberg scalar which is a gauge-like single scalar. If NGB is undistinguishable boson one can understand now its allocation which comprises of 8 (eight) NGBs for $V_{H''}^{(3)}(H_0'')$ and 8 (eight) vanished NGBs for $V_C^{(3)}(H'', H, H')$ with $H'' \equiv H_i''$ or $H'' \equiv H_0''$, and 2 (two) NGBs for $H_H'', i = 1, 2$.

The co-existence of 2 (two) PNB Higgses and a gauge-like Heisenberg scalar in a newly-formed scalar yields the exotic 3-scalar Higgs and indicates the existence of double vacua system. This is indeed what happens because NGB can create multi vacua which becomes the basis for Heisenberg commutator-uncertainty mechanism [5].

4.2.2 The gauge-like 3-component scalar

Recalling the first case above with very small α, no free PNB Higgs is produced, but only Heisenberg scalars, due to non-breaking shift symmetry. This is realized by a small change of α-value which triggers an asymptotic shift symmetry breaking, and its correspondent local gauge breaking due to $\Delta\alpha(x) \sim \alpha$, but not for shift symmetry. In the range of α-value it has been shown in the [5] that asymptotic shift α-value lies below the so-called α-gap while shift α-value above the α-gap where α-gap is proportional with VEV difference, $\Delta\alpha \sim \Delta f'$.

In this case, nature allows α-gap becomes a barrier for NGB because asymptotic shift symmetry breaking fails to trigger shift symmetry breaking which shows that a bit more significant $\Delta f'$ is tolerable in collective breaking. As a consequence, one finds the inequality below

$$\left(\pm\frac{1}{f_1'}\xi \mp \frac{1}{f_2'}\xi\right) \neq 0, \tag{99}$$

which prohibits NGB from being absorbed by $H[H^\dagger]$ and $H'[H'^\dagger]$, instead of that, it is absorbed by H'' as a newly-formed field. Eq. (95) and (96) are re-expressed accordingly as below,

$$V_{{\rm C(1)}_{\rm col}}^{(3)} = \lambda^{(3)}(2f_1'^2 f_2'^2) e^{\frac{i}{f'}\begin{pmatrix} 0\,0 & H_0'' \\ 0\,0 & \\ -H_0''^\dagger & 0 \end{pmatrix}} e^{\frac{i\Delta f'}{f'}\left(\frac{f'}{f_1'}\theta_1' + \frac{f'}{f_2'}\theta_1'^\dagger\right)}, \tag{100}$$

$$V_{{\rm C(2)}_{\rm col}}^{(3)} = \lambda^{(3)}(2f_1'^2 f_2'^2) e^{\frac{i}{f'}\begin{pmatrix} 0\,0 & H_0'' \\ 0\,0 & \\ -H_0''^\dagger & 0 \end{pmatrix}} e^{\frac{-i\Delta f'}{f'}\left(\frac{f'}{f_2'}\theta_1' + \frac{f'}{f_1'}\theta_1'^\dagger\right)}, \tag{101}$$

where H_0'', θ_1' and $\theta_1'^\dagger$ as defined in Eq. (94) and (57). Expanding $e^{\pm i\Delta f'\left(\frac{1}{f_i'}\theta_1' + \frac{1}{f_j'}\theta_1'^\dagger\right)}, i \neq j = 1, 2$, to the lowest order and summing up $V_{{\rm C(1)}_{\rm col}}^{(3)} + V_{{\rm C(2)}_{\rm col}}^{(3)} = V_{{\rm C}_{\rm col}}^{(3)}$ and taking $\frac{1}{f_1'} \sim \frac{1}{f_2'}$ one finds finally,

$$V_{{\rm C}_{\rm col}}^{(3)} = \lambda^{(3)}(4f_1'^2 f_2'^2) e^{\frac{i}{f'}\begin{pmatrix} 0\,0 & H_0'' \\ 0\,0 & \\ -H_0''^\dagger & 0 \end{pmatrix}} \tag{102}$$

which can be rewritten, after expanding the exponential function to the second order, as

$$V_{{\rm C}_{\rm col}}^{(3)} = \lambda^{(3)}\left\{4(f_1'^2 f_2'^2) + \left(\frac{f_1'^2 f_2'^2}{f'^2}\right)v''^2 + \frac{2f_1'^2 f_2'^2}{f'^2} H_0''^\dagger H_0''\right\}. \tag{103}$$

Here, one finds immediately another variant of Heisenberg scalar with the mass-squared lower than previously-shown $H_H''[H_0'']$ which is, now, for clarity labeled as H_{op}'' and shown as below,

$$m_{H_{op}''}^2 = \frac{g'^4}{16\pi^2}\frac{\lambda^{(3)}}{f'^2}(2f_1'^2 f_2'^2)\log\left(\frac{\Lambda_{(3)}^2}{\mu_{H_{op}''}^2}\right), \tag{104}$$

where $\mathcal{O}(\mu_{H''_{op}}) \sim \mathcal{O}(100 \text{ GeV})$. The 3-component scalars can be composed from $1[2]H''_H$ and $2[1]H''_{op}$, if one assigns H''_0 for Heisenberg scalar bonded by PNB Higgses, which consist of the following combinations: $H''_{op} - H''_H - H''_{op}, H''_H - H''_{op} - H''_H, 3 - H''_{op}$ and $3 - H''_H$. The mass-squareds of these combined 3-component scalars can be expressed successively as follows,

$$m^2_{H''_{(op-H-op)}} = \frac{g'^4}{16\pi^2}\frac{\lambda^{(3)}}{f'^2}\Big\{(4f_1'^2 f_2'^2) + (f_1'^4 + f_2'^4)\Big\}\log\left(\frac{\Lambda^2_{(3)}}{\mu^2_{H''_{(op-H-op)}}}\right), \tag{105}$$

$$m^2_{H''_{(H-op-H)}} = \frac{g'^4}{16\pi^2}\frac{\lambda^{(3)}}{f'^2}\Big\{(2f_1'^2 f_2'^2) + 2(f_1'^4 + f_2'^4)\Big\}\log\left(\frac{\Lambda^2_{(3)}}{\mu^2_{H''_{(H-op-H)}}}\right), \tag{106}$$

$$m^2_{H''_{3op}} = \frac{g'^4}{16\pi^2}\frac{\lambda^{(3)}}{f'^2}\Big\{6(f_1'^2 f_2'^2)\Big\}\log\left(\frac{\Lambda^2_{(3)}}{\mu^2_{H''_{3op}}}\right), \tag{107}$$

$$m^2_{H''_{3H}} = \frac{g'^4}{16\pi^2}\frac{\lambda^{(3)}}{f'^2}\Big\{3(f_1'^4 + f_2'^4)\Big\}\log\left(\frac{\Lambda^2_{(3)}}{\mu^2_{H''_{3H}}}\right), \tag{108}$$

where $\mathcal{O}(\mu_{H''_{(H-2op)}}) \sim \mathcal{O}(\mu_{H''_{(2H-op)}}) \sim \mathcal{O}(\mu_{H''_{3op}}) \sim \mathcal{O}(\mu_{H''_{3H}}) \sim \mathcal{O}(100 \text{ GeV})$. These massive 3-component scalars have 3 (three) degrees of freedom i.e. 3 free and 3 bonded degrees of freedom. Each massive scalar has 2 (two) bonded and 1 (one) free degree of freedom, these look like a triangle with one free degree of freedom at each angle point as shown in Appendix B, and are named as triatomic molecule-like rather weakly-coupled gauge-like 3-component scalars.

Different variants can be obtained if every 2 (two) H''_{op}, H''_H form a scalar-pair prior to the unification with another free Heisenberg scalar where each $H''_{op}[H''_H]$ in the scalar-pair has 2 (two) bonded and 1 (one) free degree of freedom where the free ones bind with 2 (two) degrees of freedom of the stand-alone Heisenberg scalar leaving only 1 (one) free degree of freedom. This configuration gives a Higgs-like 3-component scalar with 1 (one) free degree of freedom and is named as 3-component pseudo Heisenberg scalar whose masses are exactly the same with their gauge-like counterparts as already shown in Eq. (107) and (108). These 3-component pseudo Heisenberg scalars are strongly-coupled with 2 bonded degrees of freedom in the scalar-pair and labeled as $H''_H - 2H''_{op}, 2H''_H - H''_{op}, 2H''_{op} - H''_{op}$ and $2H''_H - H''_H$ respectively. For the sake of clarity one names these pseudo Heisenberg scalars as tritonic nuclear-like, Higgs-like 3-component scalar and presents the schematic drawings as depicted in Appendix B.

Next, a brief discussion on the shift and asymptotic shift symmetry breaking, the relationship between the two and its effect on the global and local symmetry due to global-local correspondence is given in the separate paper [5].

5 Phenomenological Aspects

5.1 The Order of SU(3)×SU(3) cut-off scale

Some phenomenological aspects within the current model are briefly examined. That is, the order estimations for the cut-off scale.

First of all, let us perform the order estimation for the cut-off scale $\Lambda_{(3)}$. The contribution of quadratically divergent one-loop diagram to the Higgs mass in the conventional Simplest Little Higgs SU(3)×SU(3)×U(1) is given by the first part of Eq. (109) [30,32] and, after performing a little calculation, one finds the second part at the righthand as

$$\sim \frac{g'^2}{16\pi^2}\left(\Lambda^{(0)}\right)^2 \mathrm{Tr}\left(\phi^{(1)\dagger}\phi^{(1)} + \phi^{(2)\dagger}\phi^{(2)}\right) \text{ or } \sim \frac{g'^2}{16\pi^2}\left(\Lambda^{(0)}\right)^2 \times 3, \tag{109}$$

where $\Lambda^{(0)}$ is the cut-off scale at the conventional Simplest Little Higgs. Hence, the cut-off scale at the present theory can be estimated to be $\Lambda_{(3)} > \sqrt{3}\Lambda^{(0)}$. On the other hand, from [30,32] the cut-off scale $\Lambda^{(0)}$ is around 10 TeV with VE$V f^{(0)} \sim 1$ TeV. It is also related to the SM cut-off $M_{\mathrm{weak}} \sim 80$ GeV through $\Lambda^{(0)} \sim 4\pi f^{(0)} \sim (4\pi)^2 M_{\mathrm{weak}}$. From above one can set, $\sqrt{3}\Lambda^{(0)} < \Lambda_{(3)} < 4\pi f'$, then the cut-off scale $\Lambda_{(3)}$ within the present model, taking the upper limit, ~ 100 TeV for $f' \sim 10$ TeV. Therefore, the cut-off scale and VEV at the present scenario is higher than the ones in the conventional Simplest Little Higgs scenario.

5.2 Higgs spectrum and masses

Three Higgs bosons emerge as the beyond-SM Higgses of SU(3) level with masses shown in Eq. (79), based on one-by-one breaking, for two light Higgs bosons and Eq. (91), (105), (106), based on collective breaking, for light 3-scalar Higgs bosons. If one sets $f'_1 \sim 4.0$ TeV, and $f'_2 \sim 5.0$ TeV, $g' = 0.6$ and $\lambda^{(3)} \sim 0.1$ with cut-off scale $\Lambda_{(3)} \sim 10$ TeV as above and $\mu_{H''} \sim 100$ GeV then masses of Higgses are 79 GeV for light Higgses H''_1 and H''_2 while Eq. (91) gives the rather strongly-coupled 3-scalar Higgs boson $H''_i - H''_0 - H''_i$, $m_{H''_{col}} \sim 139$ GeV with Heisenberg scalars, $m_{H''_H} \sim 114$ GeV and $m_{H''_{op}} \sim 79$ GeV. It means that the gauge-like single scalars have the masses of ~ 79 GeV and ~ 114 GeV at this level. On the other side Eq. (105) and (106) provide masses of strongly-coupled Higgs-like pseudo Heisenberg scalar $H''_{op} - H''_H - H''_{op}$ and $H''_H - H''_{op} - H''_H$, for the first is $m_{H''_{op-H-op}} \sim 160$ GeV and the second $m_{H''_{H-op-H}} \sim 180$ GeV. One also obtains rather weakly-coupled gauge-like scalars, $3 - H''_{op}$ and $3 - H''_H$ with masses $m_{H''_{3op}} \sim 137$ GeV and $m_{H''_{3H}} \sim 197$ GeV respectively.

On the other hand if VEVs are set at $f'_1 = 16$ TeV, $f'_2 = 20.0$ TeV and $\Lambda_{(3)} \sim 100$ TeV and other parameters remain the same then one finds mass of strongly-coupled 3-scalar heavy Higgs to be 567 GeV, still below unitary constraint ~ 700 GeV while Eq. (79) excludes the light Higgs boson $m_{H''_i} \sim 392$ GeV. This reconfirms that Eq. (79) is valid for light Higgs while Eq. (91) for both light and heavy 3-scalar Higgses with the excluded region in-between the light and the heavy lies, based on latest LHC data, in the interval $145 - 466$ GeV. The large Heisenberg mass is found to be 411 GeV with VEV $\mathcal{O}(f'_i) \sim \mathcal{O}(10$ Te$V)$ which is excluded from free Higgs region but, of course, included in free gauge-like scalar region (up to 1.5 TeV) [20]. Thus (being together with PNB), Heisenberg scalar always lives in a small confined space of 3-scalar Higgs.

Masses of molecule-like rather weakly-coupled gauge-like 3-component scalar and nuclear-like strongly-coupled 3-scalar Higgs-like scalar are basically 3 (three) times of Heisenberg scalar's mass-squared for the first and in-between 1-3 times of the mass-squared for the second. Masses of the first, $3 - H''_{op}$ and $3 - H''_H$, becomes as $m_{H''_{3op}} \sim 677$ GeV with $m_{H''_{op}} \sim 392$ GeV and $m_{H''_{3H}} \sim 712$ GeV while of the second $m_{H''_{op-H-op}} \sim 690$ GeV and $m_{H''_{H-op-H}} \sim 700$ GeV which are within the range of heavy Higgs mass. As a free particle Heisenberg scalar becomes a gauge-like single scalar with the same mass (411 GeV) but as bonded particles the light and heavy Heisenberg scalars with masses of 392 GeV and 411 GeV successively. Maximum mass of gauge-like 3-component scalar can be approximated with $f' = f'_i\sqrt{2}$ where it is found $f'_i \sim 70$ TeV which gives the required mass of 1.55 TeV. All these exotic Higgses and scalars, quite possibly of CDM relics, are Majorana-like masses [5].

6 Conclusion

The near-brane (5D, $y \sim 0$) requires AdS/CFT correspondence to remain intactly so that one finds correspondence between 5D ($y \sim 0$) local gauge and 4D global gauge symmetry in SU(6) even after dimensional deconstruction which activates the Uncertainty principle to work and underly the near-brane uncertainty correspondence i.e, another form of local-global gauge correspondence. A VEV of SU(6) is relatively large enough, it requires very small α-value so that a very small change $\Delta\alpha(x) \sim \alpha$ triggers asymptotic shift symmetry breaking without being accompanied by local gauge symmetry breaking, since Uncertainty correspondence requirement $\Delta\alpha(x) \sim \alpha$ results in a very small change $\Delta\alpha(x)$ which does not trigger local gauge symmetry breaking. The co-existence of local-global gauge symmetry proves that AdS/CFT correspondence is still valid and opens the way for dimensional deconstruction without local-global gauge symmetry breaking, (5D) SU(6) $\longrightarrow$ (4D) SU(6).

As the system of particles moves down from high to low energy level SU(6) VEV decreases as well and consequently it increases $\alpha \sim \Delta\alpha(x)$ and provides 2 (two) possibilities i.e. $\Delta\alpha(x)$ is still unsignificant or becoming significant with respect to $\alpha(x)$. The first allows SU(6) and its sextet to exist in the near-brane even into lower-near-brane. In this case SU(6) would-be Baby (Little) Higgs, under the requirement of trivial and pseudo non-trivial manners, changes into SU(6) Baby Higgs with cut off-scale $\Lambda^{\mathrm{ZP}}_{(6)}$ which is weakly-coupled. The second demands the breaking of SU(6) and its triplet-triplet splitting in the lower near-brane due to strongly-coupled SU(6) will-be-SimplestLittleHiggs, under trivial and pseudo non-trivial requirement, can not exist or last too long, instead of, it must transform itself via triplet-triplet splitting. In fact this is in line with strongly-coupled condition which demands a very high cut-off scale above $\Lambda^{\mathrm{ZP}}_{(6)}$ but it is constrained by the fact that it must exist in the lower-near-brane. The cut-off scale of SU(6) will-be-SimpletLittleHiggs would have been close to the compactification scale, $\Lambda^{\mathrm{4D}}_{(6)} \sim M_c$, but under trivial and pseudo non-trivial requirement and lower-near-brane residence, the SU(6) strongly-coupled scalar undergoes triplet-triplet splitting which establishes the cut-off scale at $\Lambda_{(3)}$ instead of $\Lambda^{\mathrm{4D}}_{(6)} \sim M_c$.

In the lower-near-brane part where it is close to the brane the VEV ($\sim$ SU(3) VEV) is much lower which provides a significant increase of α-value $\sim \Delta\alpha(x)$. This condition demands for both asymptotic shift and local gauge symmetry breaking altogether which further requires the emerging of a double vacua with one vacuum corresponds to strongly-coupled PNB Higgs field and another one to weakly-coupled Heisenberg scalar field, each field includes its derivates.

The double vacua actually forms duality with triplet-triplet splitting and reflects the local-global gauge correspondence which, in its breakings, produce both NGBs and PNB Higgs and form a Heisenberg scalar out of every one PNB Higgs and two NGBs. Heisenberg scalar becomes, together with PNB Higgs, a basic constituent of exotic Higgses and scalars.

Beside PNB Higgs and Heisenberg scalar, three families of exotic Higgses and scalars have been found i.e. 3-scalar Higgses, (Higgs-like) 3-component pseudo Heisenberg scalars, and gauge-like 3-component scalars, the first two families are basically (rather) strongly-coupled and the second one family is rather weakly-coupled. PNB Higgses tend to be light Higgses with the masses < 145 GeV, while 3-scalar Higgses tend to be heavy Higgses with the range of masses 466 GeV $< m_{H''} <$ 700 GeV. Spectrum of mass for Heisenberg scalar, 3-component pseudo Heisenberg scalar and gauge-like 3-component scalar altogether covers almost continuous range of mass up to 1.55 TeV at $f' \sim \Lambda_{(3)} \sim 100$ TeV. The emerging of exotic Higgses and scalars above increase the probability for its detection, and in parallel, pose a new challenge to LHC to unveil its cover.

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Appendix

A The 5-D Model with Trivial and Pseudo Non-trivial Breakings

A.1 Trivial and pseudo non-trivial pattern

Recalling the Scherk-Schwarz mechanism on orbifold S^1/Z_2, i.e. Eq. (1), with the twist operator T_g is defined in a way such that Eq. (3) is satisfied. There are three possibilities:

i) $\omega = 0$ and $\{Q, Z_2\} \neq 0$.
This is the no-twist condition ($T_g = 1$) without any broken part. All generators belong to $[Q', Z_2]$, *i.e.* the symmetry is conserved.

ii) $\omega \neq 0$ and $\{Q, Z_2\} = 0$.
This obviously provides the twisted condition ($T_g \neq 1$) with some broken parts $Q = \lambda^{\hat{a}}$, and a non-zero VEV along the Q direction, $\langle A_y^{\hat{a}} \lambda^{\hat{a}} \rangle = Q \langle A_y^Q \rangle$. This requires local gauge symmetry breaking which provides a non-trivial condition and facilitates for Hosotani mechanism. Special condition is obtained when $Q = 0$ which means no generator is broken but there is a breaking *i.e* symmetry breaking is realized as twisted field but gauge symmetry is intact (trivial condition).

iii) $\omega = 0$ and $\{Q, Z_2\} = 0$, but $[Q', U] = 0$.
For $\omega = 0$, this reflects the no-twist condition but with some broken parts (non-trivial). For $\omega \neq 0$, in this case, the special condition ($Q = 0$) is applied here, so that gauge symmetry is not broken and inducing the periodic fields with a single-value, i.e. the massless PNBs. Here one has the unconventional Higgs-like mechanism due to zero commutator. Special condition, the unbroken generator Q' consisting of all generators, dictates the same states as in ii), here (pseudo non-trivial condition), which facilitates Little-like Higgs.

The orbifold breaking also splits the parity of gauge field A_M^A into even and odd parities. The unbroken 4D gauge bosons A_μ^a and the broken extra-dimensional gauge bosons $A_y^{\hat{a}}$ are even function with zero-modes, while $A_\mu^{\hat{a}}$ and A_y^a are odd function without zero-mode.

Due to orbifold singular points the parity operator Z_2 which operates at each singular point is labelled as $Z_2^{(0)}$ for $y = 0$ and $Z_2^{(1)}$ for $y = \pi R$ and the following relation holds: $U = Z_2^{(0)} Z_2^{(1)}$.

B θ-matrix and schematic drawing for exotic Higgses and scalars

B.1 The θ-matrix identifier for classifying exotic Higgses and scalars

We recall Simplest Little-like Higgs in Eq. (37) of which the PNB $\theta_i', i = 1, 2$ is rewritten, making use the so-called θ-matrix identifiers, as below

$$[H_{\rm pnb}] = \begin{pmatrix} 0 & 0 & \\ 0 & 0 & H' \\ & H^\dagger & 0 \end{pmatrix}, \quad [H_{\rm pnb}^\dagger] = \begin{pmatrix} 0 & 0 & \\ 0 & 0 & H \\ & H'^\dagger & 0 \end{pmatrix} \tag{110}$$

with SU(3) PNB θ_i' as,

$$\theta_1' = \theta_{\rm pnb}' = \frac{1}{f'}[H_{\rm pnb}], \quad \theta_2' = \theta_{\rm pnb}'^\dagger = \frac{1}{f'}[H_{\rm pnb}^\dagger] \tag{111}$$

One notices that in both breaking patterns, the one-by-one and the collective ones, a new field H'' emerges in replace of H and H'. Utilizing this property one can express the results of both breaking patterns *i.e.* $H_i'', i = 1, 2$ and $H_{\rm col}''$ in Eq. (79) and Eq. (91), in the θ-matrix as follows,

$$\begin{pmatrix} 0 & 0 & \\ 0 & 0 & (H_1'') \\ (H_1''^\dagger) & & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & \\ 0 & 0 & (H_2'') \\ (H_2''^\dagger) & & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 0 & \\ 0 & 0 & (H_i'') \\ (H_i''^\dagger) & & H_0'' \end{pmatrix}, \quad i = 1, 2, \tag{112}$$

for one-by-one and collective breaking successively where for the first H_0'' is massless while for the second H_0'' is massive. Eq. (112) with $H_i'' \sim 0, H_i'' = (H' - H)_i, i = 1, 2$, shows strongly for the existence of global symmetry as SU(2)×U(1) for the upper side and SU(2) for the lower side of Eq. (112). Nevertheless the collective product, that is 3-scalar (component) Higgs (scalar), is reflected quite well by Eq. (112) the righthand side.

Consequently there are two types of Higgses, the PNB and CDM Higgses, which live in the realm of ordinary matter and dark matter successively. One write the following θ-matrix equation,

$$[H] = [H_{\rm pnb}] + [H_{\rm cdm}], \; [H^\dagger] = [H_{\rm pnb}^\dagger] + [H_{\rm cdm}^\dagger], \tag{113}$$

where it is clear that $[H_{\rm cdm}^\dagger] = [H_{\rm cdm}] = \begin{pmatrix} 0 & & \\ & 0 & \\ & & H_0'' \end{pmatrix}$. For the atom-like rather strongly-(weakly-)coupled 3-scalar Higgses one can define $H_1''^\dagger = H_1'^\dagger - H_1^\dagger$ and $H_2''^\dagger = H_2'^\dagger - H_2^\dagger$ successively and write the θ-matrix respectively as,

$$\begin{pmatrix} 0 & 0 & \\ 0 & 0 & (H_1''^\dagger) \\ (H_1'') & & H_0'' \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 0 & \\ 0 & 0 & (H_2'') \\ (H_2''^\dagger) & & H_0'' \end{pmatrix}, \tag{114}$$

while for rather weakly-coupled 3-component gauge-like scalar one has 3 (three) Heisenberg scalars, as shown at the lefthand side, below

$$[H_{cdm}] = \begin{pmatrix} H_H'' & 0 & 0 \\ 0 & H_H'' & 0 \\ 0 & 0 & H_H'' \end{pmatrix}, \quad [H_{cdm}] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & H_H'' \end{pmatrix}, \tag{115}$$

Eq. (115) righthand shows a single Heisenberg scalar, or a gauge-like single scalar.

B.2 Schematic Drawings for Massive exotic Higgses and scalars with free and bonded degrees of freedom

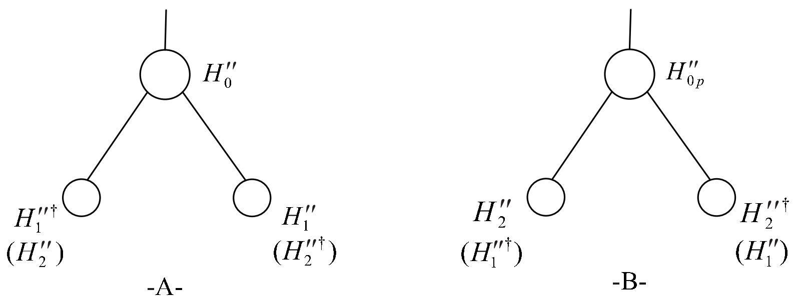

Figure 1: A. Heliumic atom-like rather strongly-coupled 3-scalar Higgs (1 free, 2 bonded degrees of freedom), B. Heliumic atom-like rather weakly-coupled 3-scalar Higgs (1 free, 2 bonded degrees of freedom).

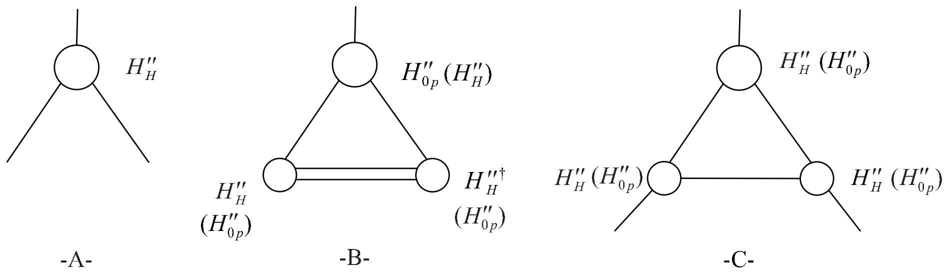

Figure 2: A. Heisenberg scalar (3 free degrees of freedom), B. Nuclear-like strongly-coupled 3-scalar Higgs-like scalar (1 free, 4 bonded degrees of freedom) or 3-component pseudo Heisenberg scalar, C. Rather weakly-coupled molecule-like 3-component scalar (3 free, 3 bonded degrees of freedom).

C Splitting potential of exotic scalar

C.1 Quadratic-based and quadratically-repeated quartic Terms of Potential $V^{(6)}_{y\mathbf{NP}}$

$$V^{(6)}_{y\mathrm{NP}(i=j)} = V^{(6)}_{y\mathrm{NP}(0)} + V^{(6)}_{y\mathrm{NP}(1)} + V^{(6)}_{y\mathrm{NP}(2)} = \lambda^{(6)}_{y\mathrm{NP}}\big(\widetilde{\Phi}^{(i)'\dagger}_{+}\widetilde{\Phi}^{(i)'}_{+}\big)^2, \quad i=1,2. \tag{116}$$

$$V^{(6)}_{y\mathrm{NP}(0)} = f_1'^4 + f_2'^4. \tag{117}$$

$$2\big(\phi_0^{(1)\dagger}\phi_0^{(1)}\big)\phi^{(1)\dagger}\phi^{(1)} = 2(f_1')^4 + \frac{f_1'^2 f_2'^2}{2f'^2}\upsilon''^2 + \frac{f_1'^2 f_2'^2}{f'^2}H''^\dagger H'',$$

$$\begin{aligned}\big(\phi^{(1)\dagger}\phi^{(1)}\big)^2 &= \Big(f_1'^4 + \frac{f_2'^4}{16f'^4}(\upsilon'')^4 + \frac{f_1'^2 f_2'^2}{2f'^2}(\upsilon'')^2\Big) + \Big(\frac{f_1'^2 f_2'^2}{f'^2} + \frac{(f_2')^4}{4f'^4}(\upsilon'')^2\Big)H''^\dagger H'' \\ &\quad + \frac{(f_2')^4}{4f'^4}(H''^\dagger H'')^2,\end{aligned}$$

$$V^{(6)}_{y\mathrm{NP}(1)} = \Big\{3f_1'^4 + \frac{(f_2')^4}{16f'^4}(\upsilon'')^4 + \frac{f_1'^2 f_2'^2}{f'^2}(\upsilon'')^2\Big\} + \Big(\frac{2f_1'^2 f_2'^2}{f'^2} + \frac{(f_2')^4}{4f'^4}(\upsilon'')^2\Big)H''^\dagger H'' + \frac{(f_2')^4}{4f'^4}(H''^\dagger H'')^2. \tag{118}$$

$$2\phi^{(2)\dagger}\phi^{(2)}\big(\phi_0^{(2)\dagger}\phi_0^{(2)}\big) = 2(f_2')^4 + \frac{f_1'^2 f_2'^2}{2f'^2}(\upsilon'')^2 + \frac{f_1'^2 f_2'^2}{f'^2}H''^\dagger H'',$$

$$\begin{aligned}\big(\phi^{(2)\dagger}\phi^{(2)}\big)^2 &= \Big(f_2'^4 + \frac{f_1'^4}{16f'^4}(\upsilon'')^4 + \frac{f_1'^2 f_2'^2}{2f'^2}(\upsilon'')^2\Big) + \Big(\frac{f_1'^2 f_2'^2}{f'^2} + \frac{(f_1')^4}{4f'^4}(\upsilon'')^2\Big)H''^\dagger H'' \\ &\quad + \frac{(f_1')^4}{4f'^4}(H''^\dagger H'')^2,\end{aligned}$$

$$V^{(6)}_{y\mathrm{NP}(2)} = \Big\{3f_2'^4 + \frac{(f_1')^4}{16f'^4}(\upsilon'')^4 + \frac{f_1'^2 f_2'^2}{f'^2}(\upsilon'')^2\Big\} + \Big(\frac{2f_1'^2 f_2'^2}{f'^2} + \frac{(f_1')^4}{4f'^4}(\upsilon'')^2\Big)H''^\dagger H'' + \frac{(f_1')^4}{4f'^4}(H''^\dagger H'')^2. \tag{119}$$

$$\begin{aligned}V^{(6)}_{y\mathrm{NP}(i=j)} = \lambda^{(6)}_{y\mathrm{NP}}\Bigg[&\Big\{4(f_1')^4 + 4(f_2')^4 + \frac{2f_1'^2 f_2'^2}{f'^2}\upsilon''^2 + \frac{(f_1')^4+(f_2')^4}{16(f')^4}(\upsilon'')^4\Big\} \\ &+ \Big\{\frac{4f_1'^2 f_2'^2}{f'^2} + \frac{(f_1')^4+(f_2')^4}{4(f')^4}\upsilon''^2\Big\}H''^\dagger H'' + \frac{(f_1')^4+(f_2')^4}{4(f')^4}(H''^\dagger H'')^2\Bigg]\end{aligned} \tag{120}$$

C.2 Product-based (fully cyclical and serially-repeated) quartic terms of potential $V^{(6)}_{y\mathbf{NP}}$

$$V^{(6)}_{y\mathrm{NP}(i\neq j)} = \big(\widetilde{\Phi}^{(1)'\dagger}_{+}\widetilde{\Phi}^{(2)'}_{+}\big)\big(\widetilde{\Phi}^{(2)'\dagger}_{+}\widetilde{\Phi}^{(1)'}_{+}\big) + \big(\widetilde{\Phi}^{(2)'\dagger}_{+}\widetilde{\Phi}^{(1)'}_{+}\big)\big(\widetilde{\Phi}^{(1)'\dagger}_{+}\widetilde{\Phi}^{(2)'}_{+}\big) = V^{(6)}_{y\mathrm{NP}1(i\neq j)} + V^{(6)}_{y\mathrm{NP}2(i\neq j)}. \tag{121}$$

$$\begin{aligned}V^{(6)}_{y\mathrm{NP}1(i\neq j)} &= \big(\widetilde{\Phi}^{(1)'\dagger}_{+}\widetilde{\Phi}^{(2)'}_{+}\big)\big(\widetilde{\Phi}^{(2)'\dagger}_{+}\widetilde{\Phi}^{(1)'}_{+}\big) \\ &= \big(\phi_0^{(1)\dagger}\phi^{(2)}\big)\big(\phi^{(2)\dagger}\phi_0^{(1)}\big) + \big(\phi_0^{(1)\dagger}\phi^{(2)}\big)\big(\phi_0^{(2)\dagger}\phi^{(1)}\big) + \big(\phi^{(1)\dagger}\phi_0^{(2)}\big)\big(\phi^{(2)\dagger}\phi_0^{(1)}\big) + \big(\phi^{(1)\dagger}\phi_0^{(2)}\big)\big(\phi_0^{(2)\dagger}\phi^{(1)}\big) \\ &= (f_1'^2 f_2'^2)e^{\frac{if_1'}{f_2'f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^\dagger & 0\end{pmatrix}} + V^{(3)'}_{\mathrm{C}(1)} + V^{(3)'}_{\mathrm{C}(2)} + (f_1'^2 f_2'^2)e^{\frac{if_2'}{f_1'f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^\dagger & 0\end{pmatrix}},\end{aligned} \tag{122}$$

$$
\begin{aligned}
V^{(6)}_{y\mathrm{NP}2(i\neq j)} &= \big(\widetilde{\Phi}_{+}^{(2)'\dagger}\widetilde{\Phi}_{+}^{(1)'}\big)\big(\widetilde{\Phi}_{+}^{(1)'\dagger}\widetilde{\Phi}_{+}^{(2)'}\big)\\
&= \big(\phi_0^{(2)\dagger}\phi^{(1)}\big)\big(\phi^{(1)\dagger}\phi_0^{(2)}\big) + \big(\phi_0^{(2)\dagger}\phi^{(1)}\big)\big(\phi_0^{(1)\dagger}\phi^{(2)}\big) + \big(\phi^{(2)\dagger}\phi_0^{(1)}\big)\big(\phi^{(1)\dagger}\phi_0^{(2)}\big) + \big(\phi^{(2)\dagger}\phi_0^{(1)}\big)\big(\phi_0^{(1)\dagger}\phi^{(2)}\big)\\
&= (f_1'^2 f_2'^2)e^{\frac{if_1'}{f_2'f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^\dagger & 0\end{pmatrix}} + (f_1'^2 f_2'^2)e^{\left[\frac{i}{f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^\dagger & 0\end{pmatrix} + \frac{i\Delta f'}{f'}\left\{\frac{1}{f_1'}\begin{pmatrix} 0\,0 & H' \\ 0\,0 & \\ H^\dagger & 0\end{pmatrix} + \frac{1}{f_2'}\begin{pmatrix} 0\,0 & H \\ 0\,0 & \\ H'^\dagger & 0\end{pmatrix}\right\}\right]}\\
&\quad + (f_1'^2 f_2'^2)e^{\left[\frac{i}{f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^\dagger & 0\end{pmatrix} - \frac{i\Delta f'}{f'}\left\{\frac{1}{f_2'}\begin{pmatrix} 0\,0 & H' \\ 0\,0 & \\ H^\dagger & 0\end{pmatrix} + \frac{1}{f_1'}\begin{pmatrix} 0\,0 & H \\ 0\,0 & \\ H'^\dagger & 0\end{pmatrix}\right\}\right]} + (f_1'^2 f_2'^2)e^{\frac{if_2'}{f_1'f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^\dagger & 0\end{pmatrix}},
\end{aligned}
\tag{123}
$$

with $\Delta f' = f_2' - f_1', H'' = H' - H, H' \sim H (H'' \sim 0)$, one finds

$$
\begin{aligned}
\left\{\frac{1}{f_i'}\begin{pmatrix} 0\,0 & H' \\ 0\,0 & \\ H^\dagger & 0\end{pmatrix} + \frac{1}{f_j'}\begin{pmatrix} 0\,0 & H \\ 0\,0 & \\ H'^\dagger & 0\end{pmatrix}\right\} &\xrightarrow[i,j=1,2]{i\neq j} \frac{f_1'+f_2'}{f_1'f_2'}\begin{pmatrix} 0\,0 & H \\ 0\,0 & \\ H^\dagger & 0\end{pmatrix},\\
\frac{i\Delta f'}{f'}\left\{\frac{f_1'+f_2'}{f_1'f_2'}\begin{pmatrix} 0\,0 & H \\ 0\,0 & \\ H^\dagger & 0\end{pmatrix}\right\} &= \frac{-i}{f'}\frac{(f_1'^2 - f_2'^2)}{f_1'f_2'}\begin{pmatrix} 0\,0 & H \\ 0\,0 & \\ H^\dagger & 0\end{pmatrix},\\
-\frac{i\Delta f'}{f'}\left\{\frac{f_1'+f_2'}{f_1'f_2'}\begin{pmatrix} 0\,0 & H \\ 0\,0 & \\ H^\dagger & 0\end{pmatrix}\right\} &= \frac{i}{f'}\frac{(f_1'^2 - f_2'^2)}{f_1'f_2'}\begin{pmatrix} 0\,0 & H \\ 0\,0 & \\ H^\dagger & 0\end{pmatrix},
\end{aligned}
$$

$$
V^{(3)'}_{\mathrm{C}(1)} = (f_1'^2 f_2'^2)e^{-\frac{i}{f'}\frac{(f_1'^2-f_2'^2)}{f_1'f_2'}\begin{pmatrix} 0\,0 & H \\ 0\,0 & \\ H^\dagger & 0\end{pmatrix}}e^{\frac{i}{f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^\dagger & 0\end{pmatrix}}, \quad V^{(3)'}_{\mathrm{C}(2)} = (f_1'^2 f_2'^2)e^{\frac{i}{f'}\frac{(f_1'^2-f_2'^2)}{f_1'f_2'}\begin{pmatrix} 0\,0 & H \\ 0\,0 & \\ H^\dagger & 0\end{pmatrix}}e^{\frac{i}{f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^\dagger & 0\end{pmatrix}},
\tag{124}
$$

$$
\begin{aligned}
V^{(6)}_{y\mathrm{NP}1(i\neq j)} + V^{(6)}_{y\mathrm{NP}2(i\neq j)} &= \Big\{V^{(3)}_{\mathrm{H}(1)} + V^{(3)}_{\mathrm{H}(2)}\Big\} + 2\Big\{V^{(3)'}_{\mathrm{C}(1)} + V^{(3)'}_{\mathrm{C}(2)}\Big\}\\
&= \left\{2(f_1'^2 f_2'^2)e^{\frac{if_1'}{f_2'f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^\dagger & 0\end{pmatrix}} + 2(f_1'^2 f_2'^2)e^{\frac{if_2'}{f_1'f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^\dagger & 0\end{pmatrix}}\right\} + \Big\{2V^{(3)'}_{\mathrm{C}(1)} + 2V^{(3)'}_{\mathrm{C}(2)}\Big\}.
\end{aligned}
\tag{125}
$$

Defining: $V^{(3)}_{\mathrm{C}(1)} = 2V^{(3)'}_{\mathrm{C}(1)}, V^{(3)}_{\mathrm{C}(2)} = 2V^{(3)'}_{\mathrm{C}(2)}$ and expanding exponential term to 2nd-order one finds,

$$
V^{(3)}_{\mathrm{H}(1)} = 2(f_1'^2 f_2'^2) + \frac{f_1'^4}{2f'^2}\upsilon''^2 + \frac{f_1'^4}{f'^2}H''^\dagger H'', \quad V^{(3)}_{\mathrm{H}(2)} = 2(f_1'^2 f_2'^2) + \frac{f_2'^4}{2f'^2}\upsilon''^2 + \frac{f_2'^4}{f'^2}H''^\dagger H'', \tag{126}
$$

and applying $f_1' \sim f_2'$ for collective breaking,

$$
V^{(3)}_{\mathrm{C}(1)} = 2(f_1'f_2')^2 e^{\frac{i}{f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^\dagger & 0\end{pmatrix}}, \quad V^{(3)}_{\mathrm{C}(2)} = 2(f_1'f_2')^2 e^{\frac{i}{f'}\begin{pmatrix} 0\,0 & H'' \\ 0\,0 & \\ -H''^\dagger & 0\end{pmatrix}} \tag{127}
$$

$$
V^{(3)}_{\mathrm{C}(1)} = V^{(3)}_{\mathrm{C}(2)} = 2(f_1'^2 f_2'^2) + \frac{f_1'^2 f_2'^2}{2f'^2}\upsilon''^2 + \frac{f_1'^2 f_2'^2}{f'^2}H''^\dagger H''. \tag{128}
$$

Thus,

$$
\begin{aligned}
V^{(6)}_{y\mathrm{NP}(i\neq j)} &= \lambda^{(6)}_{y\mathrm{NP}}\left\{4f_1'^2 f_2'^2 + \frac{f_1'^4 + f_2'^4}{2f'^2}\upsilon''^2 + \frac{f_1'^4 + f_2'^4}{f'^2}H''^\dagger H''\right\}\\
&\quad + \lambda^{(6)}_{y\mathrm{NP}}\left\{\frac{2f_1'^2 f_2'^2}{f'^2}H''^\dagger H'' + \frac{f_1'^2 f_2'^2}{f'^2}\upsilon''^2 + 4(f_1'^2 f_2'^2)\right\}.
\end{aligned}
\tag{129}
$$

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