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Grand unification theory based on SU(6) symmetry - symmetry breaking through Higgs mechanism

A. Hartanto[a] and L. T. Handoko[a,b]
[a]Department of Physics, University of Indonesia\footnote, Kampus UI Depok, Depok 16424, Indonesia
[b]Group for Theoretical and Computational Physics, Research Center for Physics, Indonesian Institute of Sciences, Kompleks Puspiptek Serpong, Tangerang 15310, Indonesia

We propose the minimal Higgs scalars to break the grand unified theory with SU(6) symmetry down to the standard model, $SU(6) \rightarrow SU(3)_C \times SU(3)_H \times U(1)_C \rightarrow SU(3)_C \times SU(2)_L \times U(1)_B \times U(1)_C \rightarrow SU(3)_C \times U(1)_{EM}$. It is found that in the minimal SU(6) GUT one should introduce at least five Higgs scalars belong to 21-, 20- and two 15- dimensional representations to realize three stages of symmetry breaking. It is shown that in the present model the seesaw mechanism should occur at the second stage of symmetry breaking.

I. INTRODUCTION

Recently, a new type of grand unified theory (GUT) with a symmetry based on the rank-5 group, SU(6) group, has been developed [1]. The model provides a new alternative for GUT model beside for instance the well-known SO(10) GUT [2]. At the same time, the model also accomodates naturally the heavy Majorana neutrinos required to enable the seesaw mechanism. On the other hand, it is also expected that higher rank than, for instance the SU(5) GUT [3], makes SU(6) GUT has more scales before breaking down to the standard model that is important to avoid too fast proton decay.

Now, we present the minimal SU(6) GUT in term of Higgs mechanism [4] to break SU(6) down to the electroweak scale to achieve a phenomenologically acceptable model. As proposed in [1}, there are two stages of symmetry breaking, that is

$$\begin{aligned} SU(6) &\rightarrow SU(3)_C \otimes SU(3)_H \otimes U(1)_C \\ &\rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_B \otimes U(1)_C , \end{aligned} \tag{1}$$

before reaching the electromagnetic scale $SU(3)_C \times U(1)_{EM}$. Here the subscripts B, C (in U(1)) and H denote the new quantum numbers in the model.

Following a requirement for the anomaly free combination of representations of fermions in SU(6) group [5], one should bring the multiplets of 2 {6} + {15} for fermions. Concerning the extended Gell-Mann Okubo relation [1], the fermions are assigned in the right-handed sextet fermion and left-handed decapentuplet as follow,

$$(\psi_1^6)_R^i = \begin{pmatrix} d_r \\ d_b \\ d_g \\ e^+ \\ \nu^C \\ N \end{pmatrix}_R , \quad (\psi_2^6)_R = \begin{pmatrix} d'_r \\ d'_b \\ d'_g \\ e'^+ \\ \nu'^C \\ N' \end{pmatrix}_R^i$$

$$(\psi^{15})_L^i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_g^C & -u_b^C & -u_r & -d_r & -d''_r \\ -u_g^C & 0 & u_r^C & -u_b & -d_b & -d''_b \\ u_b^C & -u_r^C & 0 & -u_g & -d_g & -d''_g \\ u_r & u_b & u_g & 0 & e^+ & e''^+ \\ d_r & d_b & d_g & -e^+ & 0 & N''^C \\ d''_r & d''_b & d''_g & e''^+ & -N''^C & 0 \end{pmatrix}_L^i \tag{2}$$

where i denotes the generation number and r, g, b are the colors respectively. This assignment is the most general one to accomodate 15 known fermions in the SM by inserting 12 exotic particles, indicated by prime or double prime, which have the same quantum numbers as its counterparts. L and R are the projection operators. We remark that in the present paper we adopt a general fermion assignment similar to [6] rather than the simplest one proposed in [1].

Having those multiplets at hand, we can construct the Yukawa interactions by introducing appropriate Higgs scalars.

II. YUKAWA INTERACTION

Since the fermions are in {6} and {15} representations, the scalars that can form the Yukawa couplings must belong to the tensor products,

$$\{G\} \otimes \{G\} = \{21\} \oplus \{\overline{15}\},$$
$$\{G\} \otimes \{\overline{15}\} = \{70\} \oplus \{\overline{20}\}, \quad (3)$$
$$\{\overline{15}\} \otimes \{\overline{15}\} = \{105\} \otimes \{105\} \otimes \{\overline{15}\}.$$

For the sake of simplicity, we choose the minimal dimensional representations, that is 21-, two 15- and 20-dimensional representations responsible for the first, second and third stage of symmetry breaking. As will be clarified later, these lead to the combination of Yukawa interactions,

$$\begin{aligned}\mathcal{L}_Y = \mathrm{tr}\Big[& f_u \overline{\psi^{15}}_R \Phi^{15'} \psi_L^{15} \Big] \\ & + f_\nu \overline{\psi_1^6}_R \Phi_1^{15} \psi_{1L}^6 + f_{\nu'} \overline{\psi_2^6}_R \Phi_2^{15} \psi_{2L}^6 \quad (4) \\ & + f_d \overline{\psi_1^6}_R \psi_L^{15} \Phi_1^{20} + f_{d'} \overline{\psi_2^6}_R \psi_L^{15} \Phi_2^{20} + \text{h.c.}\end{aligned}$$

to generate the massess of up-type quarks, neutrinos, down-type quarks and leptons. Here, f's are the Yukawa couplings respectively. The reader should remark that the superscripts in Higgs fields denote the original dimension of complex Higgs fields as written in Eq. (3), and do not mean its multiplets.

III. CONSTRUCTING THE HIGGS FIELDS

Following the general prescription provided in [7], we can determine each Higgs scalar mentioned above. At the first stage of symmetry breaking, SU(6) → $SU(3)_C \times SU(3)_H \times U(1)_C$, we bring 21-dimensional complex Higgs scalar. Since it generally consists of 42 real scalar fields and 35 massless gauge bosons in the SU(6) symmetry, we should then have 7 physical Higgs bosons. The 21-dimensional Higgs can be chosen to have a form,

$$\Phi^{21} = \phi_7^{21^{(0)}} I + \begin{pmatrix} \phi_1^{21^{(0)}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_2^{21^{(0)}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_3^{21^{(0)}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_4^{21^{(0)}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_5^{21^{(0)}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi_6^{21^{(0)}} \end{pmatrix} T_{35}, \quad (5)$$

where $T_i = 1/2\,\lambda i$ denotes the generator of SU(6) group [1]. The numbers inside the brackets denote the charge of Higgs fields. We have checked that in term of charge conservation there is no Yukawa interaction which is formed as combination of fermion fields with 21's. This means 21-Higgs will only break the symmetry without generating any fermion mass at its vacuum expectation value (VEV). This is the reason of disappearing 21-term in Eq. (1). Moreover it still preserves the both SU(3) symmetries due to the commutation relations: $[T_i, T_{35}] = 0 \quad (i = 1,\ldots,8 \text{ and } 27,\ldots,34)$ and also the $U(1)_C$ symmetry since $[I, T_{35}] = 0$ as expected.

For the second symmetry breaking, we take two 15-dimensional Higgs scalars. The reason for requiring two 15-dimensional Higgs scalars will be discussed soon. Let us consider the first 15-dimensional complex Higgs scalars. We then have 6 Higgs bosons from 30 real scalar fields, since there should be 17 massless gauge bosons to keep $SU(3)_C \times SU(2)_L \times U(1)_B \times U(1)_C$ symmetry and we already have 7 Higgs bosons before. Putting this 6 × 6 matrix with 6 non-zero elements inside, we have found that a consistent multiplet in term of its quantum number is,

$$\Phi^{15'} = \begin{pmatrix} 0 & 0 & 0 & \phi_1^{15'^{(0)}} & 0 & 0 \\ 0 & 0 & 0 & \phi_2^{15'^{(0)}} & 0 & 0 \\ 0 & 0 & 0 & \phi_3^{15'^{(0)}} & 0 & 0 \\ \phi_4^{15'^{(0)}} & \phi_5^{15'^{(0)}} & \phi_6^{15'^{(0)}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (6)$$

This Higgs will obviously produce the masses of up-type quarks at its VEV.

However, we are still required to generate the masses of down-type quarks, neutrinos and leptons. Concerning any possible interactions involving the sextet and pentadecuplet fermions in the Yukawa sector, it is impossible to generate the masses of down-type quarks, leptons and neutrinos simultaneously. We have checked that there is no sextet Higgs with three neutral fields inside to realize this scenario. Therefore we should consider more Higgs scalar. The most promising and minimal candidate is the second 15-dimensional Higgs. Same as before, we have 30 real scalar fields which are then deducted by 17 massless gauge bosons to arrive at 13 Higgs bosons. Concerning the charge conservation we have found that,

$$\Phi_1^{15} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_{10}^{15^{(0)}} & 0 & \phi_{11}^{15^{(0)}} \\ 0 & 0 & 0 & \phi_{11}^{15^{(0)}} & 0 & \phi_{13}^{15^{(0)}} \end{pmatrix}, \quad (7)$$

$$\Phi_2^{15} = \begin{pmatrix} 0 & 0 & 0 & \phi_2^{15^{(-1/3)}} & \phi_3^{15^{(2/3)}} & \phi_4^{15^{(-1/3)}} \\ 0 & 0 & 0 & \phi_2^{15^{(-1/3)}} & \phi_3^{15^{(2/3)}} & \phi_4^{15^{(-1/3)}} \\ 0 & 0 & 0 & \phi_2^{15^{(-1/3)}} & \phi_3^{15^{(2/3)}} & \phi_4^{15^{(-1/3)}} \\ \phi_5^{15^{(+2/3)}} & \phi_5^{15^{(+2/3)}} & \phi_5^{15^{(+2/3)}} & \phi_6^{15^{(+1)}} & \phi_7^{15^{(+2)}} & \phi_8^{15^{(+1)}} \\ \phi_9^{15^{(-1/3)}} & \phi_9^{15^{(-1/3)}} & \phi_9^{15^{(-1/3)}} & \phi_1^{15^{(0)}} & \phi_6^{15^{(+1)}} & \phi_1^{15^{(0)}} \\ \phi_{12}^{15^{(-1/3)}} & \phi_{12}^{15^{(-1/3)}} & \phi_{12}^{15^{(-1/3)}} & \phi_1^{15^{(0)}} & \phi_8^{15^{(+1)}} & \phi_1^{15^{(0)}} \end{pmatrix} . \quad (8)$$

Here we have put the neutral and charged elements in different multiplets for later convenience. As discussed later, the equality of elements (6,5) and (4,6) will enable the seesaw mechanism in the present model.

We can now accomplish the whole symmetry breaking by introducing the 20- dimensional Higgs. Using the same dimensional counting, we have 2 real scalar Higgs after deducting 40 real scalar fields with the total known Higgs bosons so far: 7 + 13 + 6 = 26 and 12 massless gauge bosons to keep $SU(3)_C \times SU(2)_L \times U(1)_B \times U(1)_C$ symmetry as well. Following the same procedure as before and concerning the last term of Yukawa interactions in Eq. (4), we put the Higgs scalar in two sextets as follow,

$$\Phi_1^{20} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \phi_1^{20^{(0)}} \\ 0 \end{pmatrix}, \quad \Phi_2^{20} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \phi_2^{20^{(0)}} \end{pmatrix}. \quad (9)$$

We remark here that 15- and 20-Higgses should be divided into two parts to incorporate the Higgses needed in the third and fifth terms in Eq. (4). However, we still have degree of freedom to choose its forms as for example written in Eqs. (7)–(9) to realize a realistic scenario such that the exotic fermion massess are much heavier than the ordinary ones, and at the same time we are able to reproduce all known gauge boson spectrums. A detail discussion on this matter can be seen in [8].

IV. CONCLUSION

We have discussed and provided explicitely the Higgs scalars required to break the SU(6) GUT down to electromagnetic scale. It has been shown that by counting the degree of freedom in each breaking level, one can determine the dimension of Higgses needed. According to this prescription, the breaking pattern of $SU(6) \to SU(3)_C \times SU(3)_H \times U(1)_C \to SU(3)_C \times SU(2)_L \times U(1)_B \times U(1)_C \to SU(3)_C \times U(1)_{EM}$ can be realized by introducing at least six Higgs scalars of 21-, two 15-, 15'- and two 20-dimensions.

On the other hand, seesaw mechanism needed to realize tiny neutrino masses occurred at the second stage of symmetry breaking through 15-Higgs. Further study on proton decay, running masses and gauge unification based on the renormalization group method within this model are still under progress and will be published subsequently.

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