Fluid QCD Approach for Quark-Gluon-Plasma in Relativistic Nuclear Collision

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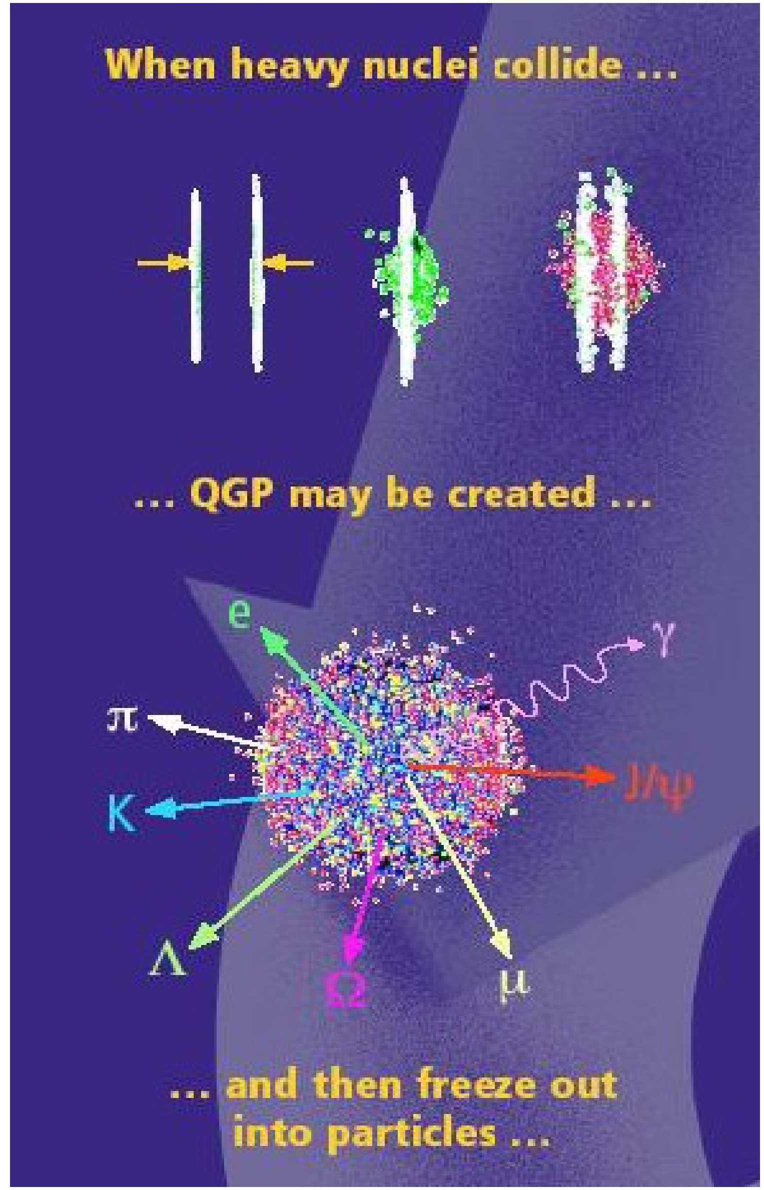
- This is the illustration of the collision between hadrons
- The collision result is the reproduction of other particles and hadrons
- Along the process of collision till the reproduction of hadrons, there is a state that exist in a very short intermediate time where the matter still cannot be considered as particle/hadron
- This state is called plasma
- Some researcher consider this state as very similar to ideal fluid, or very close to ideal fluid with small viscousity, and therefore rel.hydrodynamics is commonly used as a tool to analyse plasma.
- At our approach, we perceive plasma as consist of gluon clouds with some quarks and anti-quarks inside



Lagrangian for QCD / QGP

It is non-Abelian $SU(3)_C$

$$\mathcal{L}_{matter} = i\overline{Q}\gamma^{\mu}\partial_{\mu}Q - m_Q\overline{Q}Q$$

$$\mathcal{L}_{gauge} = -\frac{1}{4}G^{a}_{\mu\rho}G^{a\mu\rho}$$

$$\mathcal{L}_{gauge} = gJ^{a}_{\mu}U^{a\mu}$$

$$\mathcal{L}_{eff} = i\overline{Q}\gamma^{\mu}\partial_{\mu}Q - m_Q\overline{Q}Q - \frac{1}{4}G^{a}_{\mu\rho}G^{a\mu\rho} + gJ^{a}_{\mu}U^{a\mu}$$

wher

$$G^{a}_{\mu\rho} = \partial_{\mu}U^{a}_{\rho} - \partial_{\rho}U^{a}_{\mu} - gf_{abc}U^{b}_{\mu}U^{c}_{\rho}$$

and $U^a_\mu = (U^a_0, \mathbf{U}^a) \equiv u^a_\mu \phi, \qquad u_\mu \equiv \gamma^a(1, -\mathbf{v}^a)$

This velocity positions itself as a gauge field in the lagrangian. It is valid as has been shown *(IJMP-A Vol.24, Nos.18 & 19 (2009) 3630-3637)* that such kind of substitution to magneto-fluid lagrangian can produced EOM of fluid

$$\frac{\partial}{\partial t}(\gamma^a \mathbf{v^a} \phi) + \bigtriangledown(\gamma^a \phi) = -g_F \oint dx(\mathcal{J}^a_{F0} + F^a_0)$$

and then become Euler eq. when one takes its non-rel. limit.

$$\frac{\partial \mathbf{v}^a}{\partial t} + (\mathbf{v}^a \cdot \bigtriangledown)\mathbf{v}^a = -g_F \oint dx(\mathcal{J}^a_{F0} + F^a_0)_{,non-rel}$$

temporary

Review for the scenario of lagrangian approach

Approach other than hydrodynamics, means no viscous stress tensor and energy-momentum tensor embedded or be put in by hand. Where it is actually the main tools for rel.hyd. approach to extract the observables of the system of plasma.

By using a non-Abelian gauge symmetry lagrangian, it inherently assumes

- Plasma is consist of gluon cloud surrounding quarks and anti quarks matters
- In complete version there is also electromagnetic fields that interacts with gluon, quarks and anti-quarks fields. But it is too weak to be considered.

After we have the lagrangian defined, we can proceed with attempt to find the spectrum of particles with energy E and momentum p.

Using the procedure of rel.hyd. where the distribution function can be related/derived through kinetic theory, the same way can be proceed for the gluon

Distribution function

Behavior of the cloud of gluon can be described by kinetic theory

Energy-momentum tensor of kinetic theory for sta $T^{\mu\nu}_{(0)} = \int d\chi p^\mu p^\nu f_{eq}$

where $\int d\chi \equiv \frac{d^4p}{(2\pi)^3}\delta(p^\mu p_\mu - m^2)2\theta(p^0)$

For a system that consist of stable and instable p

$f = f_{eq}[1 + \delta f]$ and

$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \Pi^{\mu\nu}.$

Where $T^{\mu\nu}_{(0)} = \int d\chi p^\mu p^\nu f_{eq}$ and $\Pi^{\mu\nu} = \int d\chi p^\mu p^\nu f_{eq}\delta f.$

The calculation

- Energy-momentum tensor

$$\mathcal{T}^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \partial^\nu \varphi - g^{\mu\nu} \mathcal{L}$$

$$\begin{aligned} \mathcal{T}^{\mu\nu} &= -G^{a\mu\rho}(\partial^\nu U^a_\rho) - g^{\mu\nu}\mathcal{L} \\ &= -(\partial^\mu U^{a\rho} - \partial^\rho U^{a\mu} + g_F f^{abc} U^{b\mu} U^{c\rho})(\partial^\nu U^a_\rho) - g^{\mu\nu}\mathcal{L} \end{aligned}$$

To get the calculation close to experimental circumstance,

- Bjorken flow
- Cylindrical symmetry in transverse direction
- Cooper-Frye freeze out prescription

The metric,

$$g_{\mu\nu} = \begin{array}{c} \tau \\ \eta_s \\ r \\ \phi \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\tau^2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -r^2 \end{pmatrix}$$

$$\tau = \sqrt{t^2 - z^2}, \qquad \eta_s = \frac{1}{2} log(\frac{t+z}{t-z}),$$

$$r = \sqrt{x^2 + y^2}, \qquad \phi = \arctan(y/x)$$

η_s is space-time rapidity

τ is proper time

y is fluid rapidity

ϕ is the angle between any vector of hydrodynamics quantity (velocity,momentum, etc.) and a transverse axis

Coordinate for a free particle at a point (τ, η_s, r, ϕ)

The four-velocity is

$$\mathrm{U}^{\mu} = u^{\mu}\Phi = \gamma(1, -V)\Phi(\text{in minkowski co-ordinate})$$
$$= (\cosh\eta, u_r\cos\phi_u, u_r\sin\phi_u, \sinh\eta)\Phi$$

The four momentum is $p^{\mu} = (E, p^x, p^y, p^z)$
$$= (m_T\cosh Y, p_T\cos\phi_p, p_T\sin\phi_p, m_T\sinh Y)$$

With above formulation for all of the required
elements, and simplification (field ϕ, and velocity
gradient are assume as unity, and electromagnetic

$$\mathcal{T}^{\mu\nu} = g^{\mu\nu}(G_{a\alpha\beta}G^{a\alpha\beta} - gJ^a_\alpha U^{a\alpha})$$

$$\begin{aligned}G^a_{\mu\rho}G^{a\mu\rho} = &-2U_\rho(g^{\mu\rho}\partial^2 - \partial^\mu\partial^\rho)U_\mu + 2[(\partial_\mu U^a_\nu - \partial_\nu U^a_\mu)gf^{abc}U^{b\mu}U^{c\rho} \\ &+gf^{abc}U^b_\mu U^c_\rho(\partial^\mu U^{a\nu} - \partial^\nu U^{a\mu})] + g^2 f^{abc}f^{ade}U^{b\mu}U^{c\rho}U^d_\mu U^e_\rho\end{aligned}$$

$$\begin{aligned}U_\mu U^\mu U_\rho U^\rho = \Phi^4\Big(&\cosh^4\eta + u_r^4\cos^4\phi_u + r^8u_r^4\sin^4\phi_u + \tau^8\sinh^4\eta \\ &+2(r^4u_r^4\cos^2\phi_u\sin^2\phi_u + r^4\tau^4u_r^2\sin^2\phi_u\sinh^2\eta + \tau^4u_r^2\cos^2\phi_u\sinh^2\eta \\ &-\cosh^2\eta u_r^2\cos^2\phi_u - r^4\cosh^2\eta u_r^2\sin^2\phi_u - \tau^4\cosh^2\eta\sinh^2\eta)\Big)\end{aligned}$$

$$p^\mu d\Sigma_\mu = \Big(m_T\cosh(Y-\eta) - p_T\cos(\phi_p - \phi)\frac{\partial\tau}{\partial r}\Big)rd_r\tau d_\phi d_\eta$$

Since finite-temperature-field formation as the background is applied, and therefore τ and η

is transformed int β and z

The energy momentum tensor density is also required
to be integrated against the total volume.

$$E\frac{d^3N}{d^3p} = \int(\int f_{eq})p^{\mu}d\Sigma_{\mu} + \int(\int f_{eq}\delta f)p^{\mu}d\Sigma_{\mu}$$

$$= \int \left(m_T \cosh(Y - \tanh^{-1}(\tfrac{z}{\beta})) - p_T \cos(\phi_p - \phi)\frac{\partial\tau}{\partial r}\right)$$

$$\left(\int g^2 f^{abc} f^{ade} \Phi^4 \left(\cosh^4 \tanh^{-1}(\frac{z}{\beta}) + u_r^4 \cos^4 \phi_u + r^8 u_r^4 \sin^4 \phi_u\right.\right.$$

$$+\tau^8 \sinh^4 \tanh^{-1}(\tfrac{z}{\beta}) + 2(r^4 u_r^4 \cos^2\phi_u \sin^2\phi_u + r^4\tau^4 u_r^2 \sin^2\phi_u \sinh^2 \tanh^{-1}(\tfrac{z}{\beta})$$

$$+\tau^4 u_r^2 \cos^2\phi_u \sinh^2 \tanh^{-1}(\tfrac{z}{\beta}) - \cosh^2 \tanh^{-1}(\tfrac{z}{\beta}) u_r^2 \cos^2\phi_u$$

$$-r^4 \cosh^2 \tanh^{-1}(\tfrac{z}{\beta}) u_r^2 \sin^2\phi_u - \tau^4 \cosh^2 \tanh^{-1}(\tfrac{z}{\beta}) \sinh^2 \tanh^{-1}(\tfrac{z}{\beta}))$$

$$rdrd\phi\sqrt{\beta^2 - z^2}\Omega d\beta dz\Big)\Big) rdrd\phi z\Omega d\beta$$

$$+$$

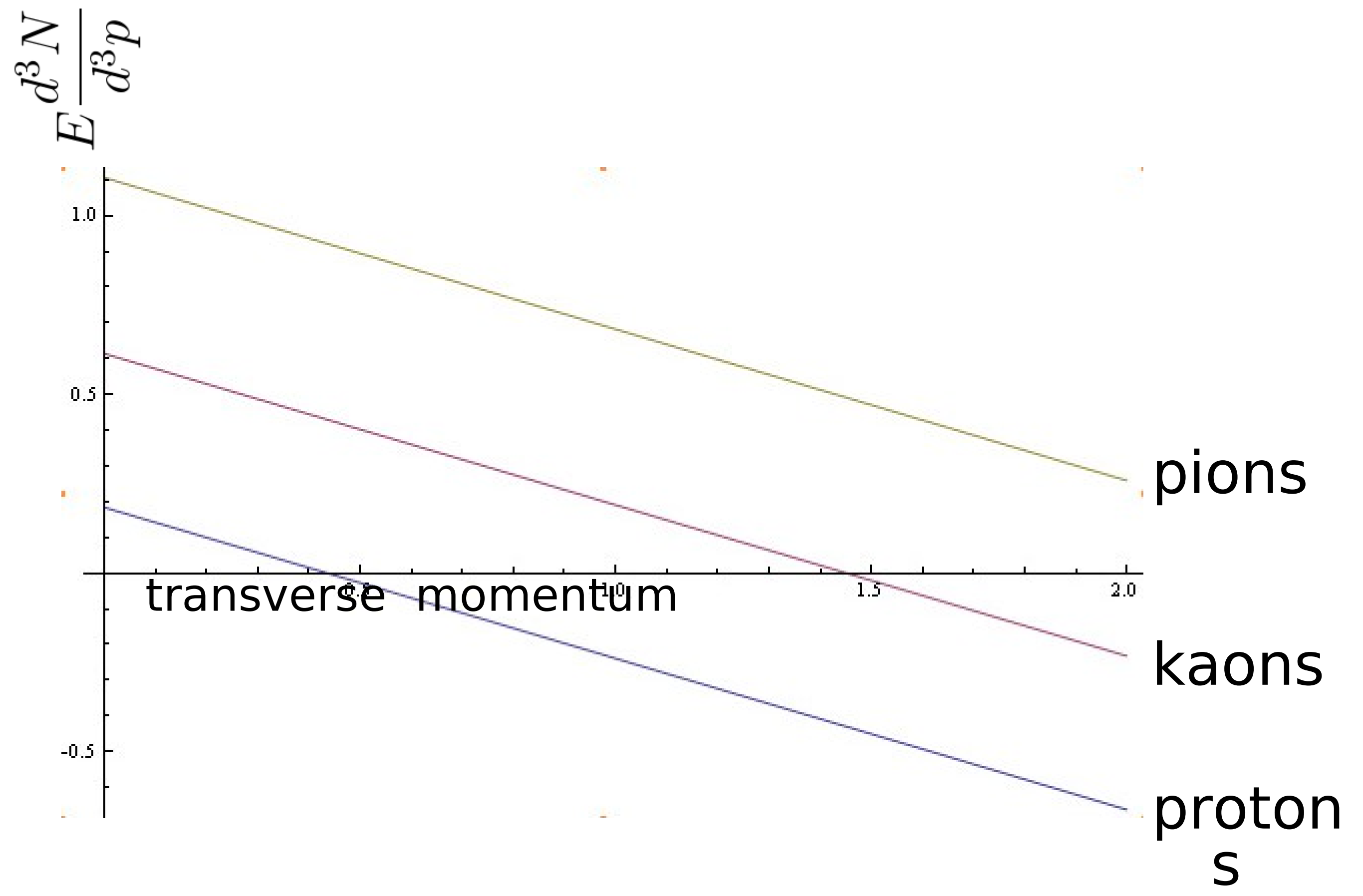
$$\int \left(m_T \cosh(Y - \tanh^{-1}(\tfrac{z}{\beta})) - p_T \cos(\phi_p - \phi)\tfrac{\partial\tau}{\partial r}\right)$$

$$\Bigg(\int \Bigg(4gT^a\Phi(m_T \cosh Y \cosh \tanh^{-1}(\frac{z}{\beta}) - p_T \cos\phi_p u_r \cos\phi_u$$

$$-r^2 p_T \sin\phi_p u_r \sin\phi_u - \tau^2 m_T \sinh Y \sinh \tanh^{-1}(\tfrac{z}{\beta}))\Big)$$

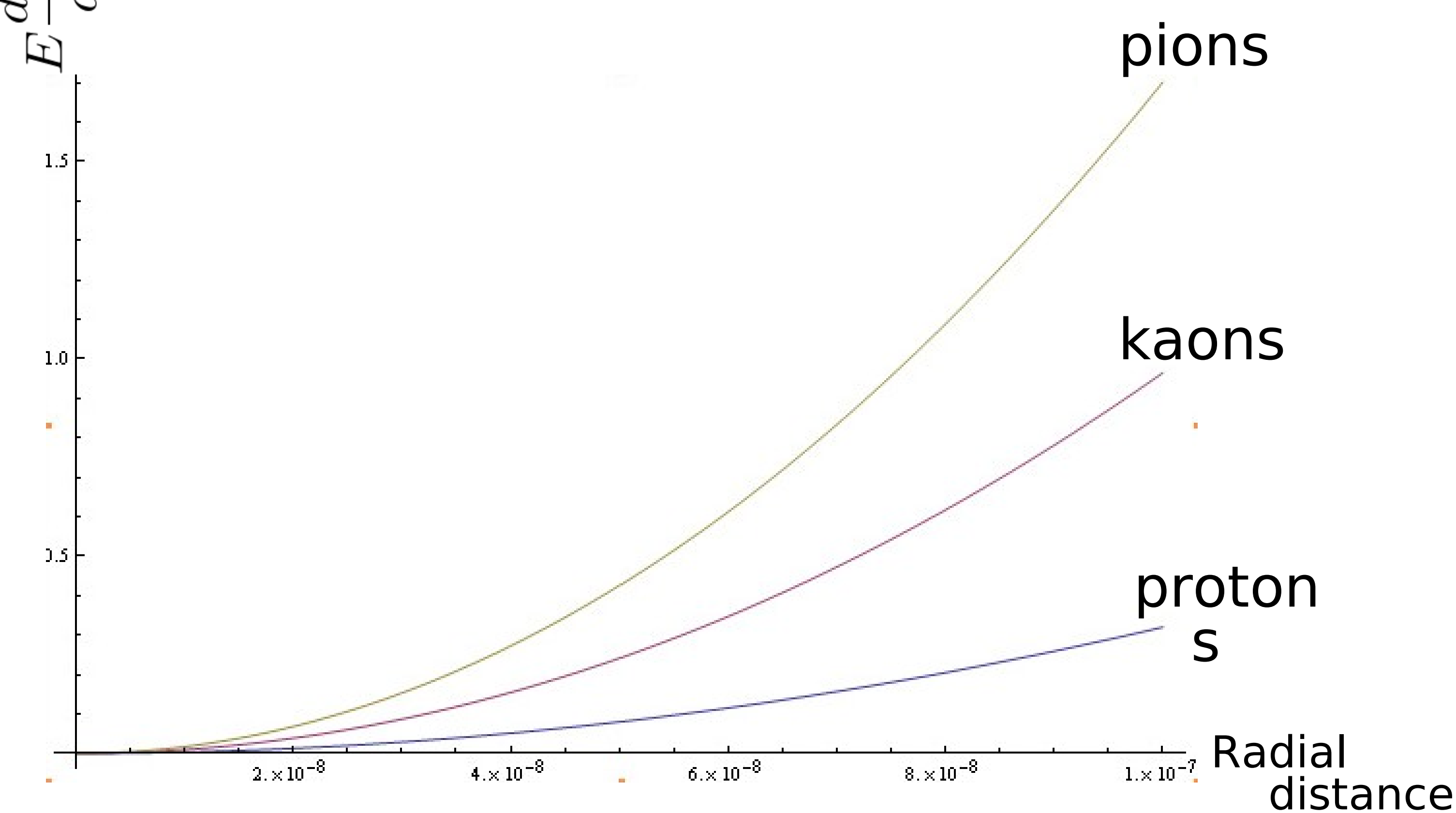
$$rdrd\phi\sqrt{\beta^2 - z^2}\Omega d\beta dz\Big) rdrd\phi z\Omega d\beta$$

This is one of observables that we have derived from the QCD lagrangian. From this equation we can produce below figures.



Particles number density against transverse
momentum

We can see that the density of plasma get higher at a farther distance from the center of collision. Perhaps we can simply explain that farther the distance means closer the time to reproduction process, and it need a more dense plasma to be formed into particle.



Particles number density against radial distance

The result of previous plots are closely similar to the result that obtained by rel.fydrodynamics approach.

Conclusion

- We have investegated the QGP based on fluid QCD as the effective theory of QCD
- We have reproduced similar result of QGP with rel.hydrosynamics approach.