Viscous Quark-Gluon Plasma Through Fluid QCD Approach

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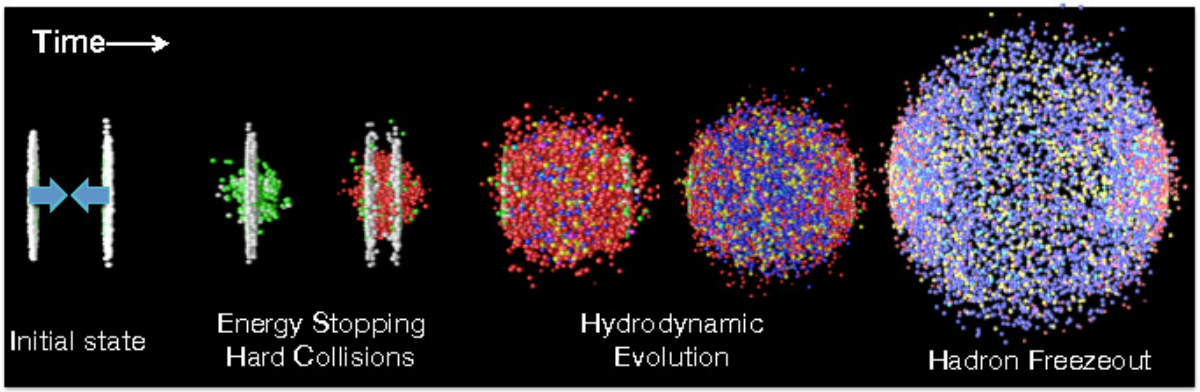
In general, plasma is defined as a fluid or gas that contain charges. The charges could be electromagnetic charge, strong charge, weak charge, or the combination of it.

Plasma is also recognized as the fourth phase of matter besides solid, liquid, and gas.

In fact, plasma dominates about 90% of both volume and mass of matter in our universe.



Heavy Ions Collission.

One of a research field that related to above definition is the research of quark-gluon plasma

Quark-gluon plasma is a phase in Quantum Chromodynamics that exist at extremely high density and temperature, which is about $2 \times 10^{13} K$.

At this phase, quarks and gluons are in a deconfined condition (asymptotic freedom).

Development in QGP Research :

Experiments for reproducing QGP through the collisions of heavy ions has been started since the beginning of 1970.

At the year 2000, it was found that macroscopically QGP behaves like ideal fluid.

(U. Heinz and R. Snellings, Ann. Rev. in Nuclear and Particle Physics 63 (2013), Collective flow and viscosity in RHIC, arXiv: nucl-th/1301.2826).

An interesting phenomena is, farther the distance from the center of collision, it shows a more dissipative behavior. This condition indicates that a construction of viscosity inside QGP theory is required (QGP based on non-ideal fluid theory).

So far, a satisfying model of viscosity distribution for QGP has not been found.
Latest theoretical result in predicting QGP viscosity : it is very small, with the lowest border $\eta/s > 1/4\pi$.

Hybridzation of particle physics and hydrodynamics :

The basic theory : QCD that extended by the unification/hybridization of various fields at Lagrangian level.

The hybridization goes in a right track as long as all the terms in lagrangian are gauge invariance.

$$\mathcal{L} \rightarrow \mathcal{T}^{\mu\nu} \rightarrow \nabla_\mu \mathcal{T}^{\mu\nu} = 0 \rightarrow EoS \rightarrow \eta?, \eta(x^\mu)?$$

Since the model for viscous QGP is build under QCD framework, and the symmetry is extended to $SU(3) \otimes SU(3)$, then the constructed viscosity is considered to be applicable for plasma with lower symmetry group.

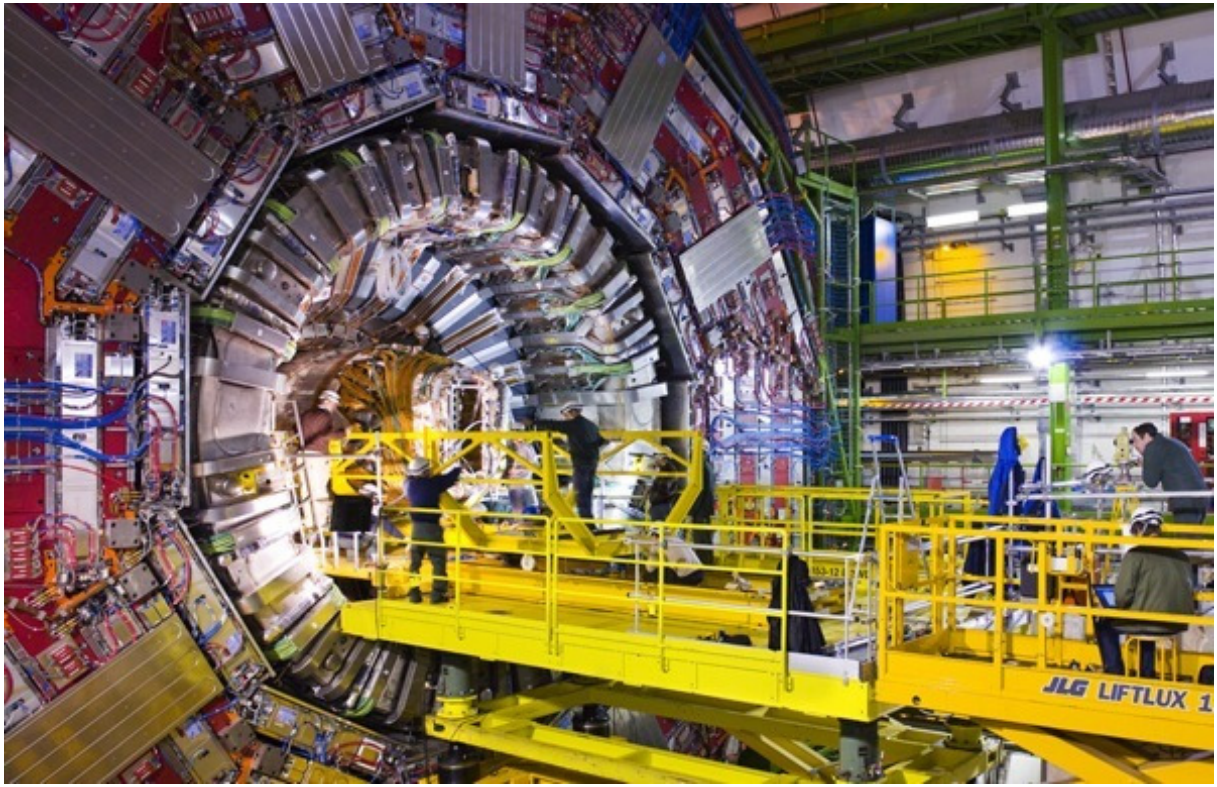
Laboratories and QGP research groups :

RHIC, The Relativistic Heavy Ion Collider, BNL, New York. The second-highest-energy heavy-ion collider ($\sim$ 500 GeV).
There are 2 QGP detectors at RHIC (STAR and PHENIX).

LHC, The Large Hadron Collider, Geneva. The world's largest and highest-energy particle accelerator. ($>$ 4 TeV per beam).
They have 7 detectors, 2 of it are used for detecting QGP.

They are ALICE (A Large Ion Collider Experiment) and ATLAS (A Toroidal LHC Apparatus).

LHC - whole sight.



LHC.

Alice.

Approaches in recent QGP researches :

QGP as a charged gas/fluid, $\rightarrow$ relativistic hydrodynamics.

P.F.Kolb, R.Rapp, Phys. Rev. C, 67, 044903 (2003)

R.Baier, P.Romatschke, Eur. Phys. J. C,51, 677 (2007)

QGP as a part of pure QCD$\rightarrow$ lattice gauge theory.

S.Gottlieb, J.Phys. Conf. Ser. **78**, 012023 (2007)

P.Petreczky, Eur. Phys. J. Special Topics **155**, (2008)

Other research with hybridization approach but not gauge invariance :

Unifying/Hybridizing quark field and fluid/flow field.

S.M. Mahajan, Phys. Rev. Lett. 90,(2003),035001, Magnetofluid Unification.

B.A. Bambah, S.M. Mahajan and C. Mukku, Phys. Rev. Lett. 97,(2006),072301,Yang-Mills Magnetofluid Unification.

Our model is build with maintaining the non-abelian gauge symmetry for all hybridized fields (the quark field, and gluon field that later on to be transformed into a flow field).

A.Sulaiman, A.Fajarudin, T.P.Djun and L.T.Handoko, *International Journal of Modern Physics A*,Vol.24, (2009),3630-3637,Magnetofluid Unification in Yang-Mills Lagrangian.

T.P.Djun, L.T.Handoko, in Proceeding of the Conferencein Honour of Murray Gell-Mann's 80th Birthday : Quantum Mechanics, Elemantary particles, Quantum

Cosmology and Complexity (2011) pp.419-425. DOI 10.1142/9789814335614040.

A. Sulaiman, T.P. Djun, L.T. Handoko, J. Theor. Comput. Stud. 5 (2006) 0401, Gauge invariant fluid lagrangian and its application to cosmology

C. S. Nugroho, A. O. Latief, T. P. Djun, L. T. Handoko, Gravitation and Cosmology 18 (2012) 32-38, Gluon matter plasma in the compact star core within fluid QCD model.

T.P. Djun, M.K.N. Patmawijaya, R. Utama, L.T. Handoko, arXiv:1303.5849, Gluonic plasma dominated early universe within fluid QCD.

From the QGP lagrangian that based on ideal fluid, it is developed to the QGP lagrangian that based on viscous fluid.

The theory and the model :

$$\begin{aligned} \mathcal{L} &= i\bar{Q}\gamma^{\mu}\partial_{\mu}Q - m_Q\bar{Q}Q - \frac{1}{4}S^{a}_{\mu\nu}S^{a_{\mu\nu}} + g_s J^{a\mu}_{F}U^{a}_{\mu} \\ &- \frac{1}{2}\eta T^{a}_{F}U^{a\mu}U^{bl}\partial_l U^{b}_{\mu}. \end{aligned} \tag{1}$$

Q is quark field.
U_μ is gauge vector field.
g_s is the strong coupling constant.
T^a is Gell-Mann matrix, the SU(3) group generator.
J^a_μ is quark current, $J^{a\mu} = \bar{Q}T^a\gamma^\mu Q$.
$S^a_{\mu\nu}$ is field strength tensor.
$S^a_{\mu\nu} = \partial_\mu U^a_\nu - \partial_\nu U^a_\mu + g_s f^{abc} U^b_\mu U^c_\nu$.
η is coefficient of shear viscosity.
f^{abc} is the structure content of SU(3) group.

To introduce fluid behavior into the equation, gluon field U^a_μ is defined as :

$$U^a_\mu = (U^a_0, \mathbf{U}^a) = u^a_\mu \phi(x) \tag{2}$$

$u^a_\mu = \gamma(1, \mathbf{v}^a)$ dan $\gamma = (1 - |\mathbf{v}^a|^2)^{1/2}$.

Since we choose to develop gluon dominated plasma, therefore the quark term is omitted.

$$\mathcal{L} = -\frac{1}{4} S^a_{\mu\nu} S^{a\mu\nu} + g_s J^a_{F\mu} U^{a\mu} - \frac{1}{2} \eta T^a_F U^{a\mu} U^{bl} \partial_l U^b_\mu. \tag{3}$$

Energi momentum tensor of QGP that based on ideal fluid :

This lagrangian represents QGP, which consist of quark and anti-quark in strong interacting gluon sea.

$$\mathcal{L} = i\bar{Q}\gamma^{\mu}\partial_{\mu}Q - m_Q\bar{Q}Q - \frac{1}{4}S^{a}_{\mu\nu}S^{a\mu\nu} + g_s J_F^{a\mu}U^{a}_{\mu}. \tag{4}$$

Q is quark (color) triplet.
U_μ is gauge vector field
g_s is the coupling constant for strong interaction

$J^a_\mu = \bar{Q} T^a \gamma_\mu Q$
T^a's is SU(3) Gell-Mann matrix
$S^a_{\mu\nu} = \partial_\mu U^a_\nu - \partial_\nu U^a_\mu + g_s f^{abc} U^b_\mu U^c_\nu$
f^{abc} is the structure constant of SU(3)

Actually quark and anti-quark also have electromagnetic interaction that arise from A_μ field of U(1) symmetry, but it is very small and can be ignored. $e/g_s = \sqrt{\alpha/\alpha_s} \rightsquigarrow O(10^{-1})$.

The mechanism : "partikel field $\longleftrightarrow$ flow field"

$$U^a_\mu = (U^a_0, \mathbf{U}^a) \equiv u^a_\mu \phi. \tag{5}$$

$u^a_\mu = \gamma_{\mathbf{v}^a}(1, \mathbf{v}^a)$ dan $\gamma_{\mathbf{v}^a} = (1 - |\mathbf{v}^a|^2)^{1/2}$.

The eom from this lagrangian is

$$\frac{\partial}{\partial t}(\gamma_{\mathbf{v}^a}\mathbf{v}^a\phi) + \nabla(\gamma_{\mathbf{v}^a}\phi) = -g_s \oint d\mathbf{x}(\mathcal{J}^a_0 + F^a_0), \tag{6}$$

$\mathcal{J}^a_\mu$ is gluon current, and F^a_μ is auxiliary function.

Take the non-rel limit of eq.(6) $\rightarrow$ one get Euler eq.

Comparing $U^a_\mu = u^a_\mu \phi$ and $U^a_\mu = \epsilon^a_\mu \phi$, where ϵ^a_μ is particle polarization vector, it shows a kind of phase transition.

$$\underbrace{hadronic\ \ state}_{\epsilon^a_\mu} \longleftrightarrow \underbrace{QGP\ state}_{u^a_\mu}$$

Through $\mathcal{T}_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}}$, and by ignoring the quark and anti-quark terms (gluon dominated QGP), one get

$$\mathcal{T}_{\mu\nu,\, non-vis} = S^a_{\mu\nu} S^{a\rho}_{\nu} - g_{\mu\nu} \mathcal{L}_g + 2 g_s J^a_\mu U^a_\nu \quad (7)$$

For the sake of simplification, gluon fields are assumed homogen for all colors $U_\mu^a = U_\mu$, $a = 1, ..., 8$,

$$\begin{aligned} \mathcal{T}_{\mu\nu,\, non-vis} &= [8 g_s f_Q m_Q \phi + g_s^2 f_g^2 \phi^4] u_\mu u_\nu \\ &- [4 g_s f_Q m_Q \phi - \frac{1}{4} g_s^2 f_g^2 \phi^4] g_{\mu\nu}. \end{aligned}$$

Energi-momentum for viscous term

$$\mathcal{T}_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_g}{\delta g^{\mu\nu}}, \tag{8}$$

$$\begin{aligned} t_{(vis)\mu\nu} = & - \eta T^a (\partial_\nu U^a_\mu + \partial_\mu U^a_\nu \\ & - U^a_\nu U^{bl} \partial_l U^b_\mu - U^a_\mu U^{bl} \partial_l U^b_\nu) \\ & + \frac{2}{3} \eta T^a \partial_l U^{al} (g_{\mu\nu} - U^b_\mu U^b_\nu). \end{aligned} \tag{9}$$

to simplify, $t_{ij} = t_{ji}$, therefore $t_{(vis)}$ becomes

$$t_{(vis)\mu\nu} = \eta T^a (\frac{14}{3} \partial^\mu u^\nu \phi - \frac{16}{3} \partial^\mu u^\nu \phi^4) \tag{10}$$

Progress

We can try to find equation of state by several ways, for example one of the most powerful tools, the conservation law, $\nabla_\mu \mathcal{T}^{\mu\nu} = 0$.

To borrow from the previous calculation result,

$$\begin{aligned} \mathcal{T}^{\mu\nu} &= [8 g_s f_Q m_Q \phi + g_s^2 f_g^2 \phi^4] u^\mu u^\nu \\ &- [4 g_s f_Q m_Q \phi - \frac{1}{4} g_s^2 f_g^2 \phi^4] g^{\mu\nu} \quad (11) \end{aligned}$$

Then,

$$\begin{aligned}
\mathcal{T}^{\mu\nu}_{total} &= \mathcal{T}^{\mu\nu} + t^{\mu\nu}_{(vis)} \\
&= [8 g_s f_Q m_Q \phi + g_s^2 f_g^2 \phi^4] u^\mu u^\nu \\
&- [4 g_s f_Q m_Q \phi - \frac{1}{4} g_s^2 f_g^2 \phi^4] g^{\mu\nu} \\
&+ \eta T^a (\frac{14}{3} \partial^\mu u^\nu \phi - \frac{16}{3} \partial^\mu u^\nu \phi^4) \quad (12)
\end{aligned}$$

Covariant derivative :
$\nabla_\mu \mathcal{T}^{\mu\nu} = \frac{1}{\sqrt{-g}} \partial_\mu \mathcal{T}^{\mu\nu} + \Gamma^\nu_{\sigma\mu} \mathcal{T}^{\mu\sigma}$.
And set $g_s f_Q m_Q = \alpha$, $g_s^2 f_g^2 = \beta$.

$$
\begin{aligned}
\nabla_\mu \mathcal{T}_1^{\mu\nu} &= \frac{1}{\sqrt{-g}}\partial_\mu[8g_s f_Q m_Q \phi + g_s^2 f_g^2 \phi^4]u^\mu u^\nu \\
&+ \Gamma^\nu_{\sigma\mu}[8g_s f_Q m_Q \phi + g_s^2 f_g^2 \phi^4]u^\mu u^\sigma. \\
&= \frac{1}{\sqrt{-g}}[8\alpha(\partial_\mu \phi) + \beta(\partial_\mu \phi^4)]u^\mu u^\nu \\
&+ \frac{1}{\sqrt{-g}}[8\alpha\phi + \beta\phi^4](\partial_\mu u^\mu)u^\nu \\
&+ \frac{1}{\sqrt{-g}}[8\alpha\phi + \beta\phi^4]u^\mu(\partial_\mu u^\nu) \\
&+ \Gamma^\nu_{\sigma\mu}([8\alpha\phi + \beta\phi^4]u^\mu u^\nu) \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
\nabla_\mu \mathcal{T}_2^{\mu\nu} &= g^{\mu\nu}\frac{1}{\sqrt{-g}}[4\alpha(\partial_\mu\phi) - \frac{1}{4}\beta(\partial_\mu\phi^4)] \\
&+ \Gamma^\nu_{\sigma\mu}[(4\alpha\phi - \frac{1}{4}\beta\phi^4)g^{\mu\nu}] \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
\nabla_\mu \mathcal{T}_3^{\mu\nu} &= \frac{14}{3\sqrt{-g}}\eta T[2(\partial_\mu\partial^\mu u^\nu)\phi + \partial^\mu u^\nu(\partial_\mu\phi)] \\
&+ \Gamma^\nu_{\sigma\mu}(\frac{14}{3}\eta T\partial^\mu u^\sigma\phi) \\
&= \frac{14}{3\sqrt{-g}}\eta T[2(\partial_\mu\partial^\mu)u^\mu\phi + g^{\mu\nu}\partial^\mu u_\mu(\partial_\mu\phi)] \\
&+ \Gamma^{\nu\mu}_{\mu} g_{\mu\sigma}(\frac{14}{3}\eta T g^{\mu\sigma}\partial^\mu u_\sigma\phi) \\
&= \frac{14}{3\sqrt{-g}}\eta T[2(\partial_\mu\partial^\mu)u^\mu\phi + g^{\mu\nu}\partial^\mu u_\mu(\partial_\mu\phi)] \\
&+ 4\Gamma^{\nu\mu}_{\mu}(\frac{14}{3}\eta T\partial^\mu u_\mu\phi) \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
\nabla_\mu \mathcal{T}_4^{\mu\nu} &= \frac{16}{3\sqrt{-g}}\eta T[2(\partial_\mu\partial^\mu u^\nu)\phi^4 + \partial^\mu u^\nu(\partial_\mu\phi^4)] \\
&+ \Gamma^\nu_{\sigma\mu}(\frac{16}{3}\eta T\partial^\mu u^\sigma\phi^4) \\
&= \frac{16}{3\sqrt{-g}}\eta T[2(\partial_\mu\partial^\mu u^\nu)\phi^4 + g^{\mu\nu}\partial^\mu u_\mu(\partial_\mu\phi^4)] \\
&+ 4\Gamma^{\nu\mu}_{\mu}(\frac{16}{3}\eta T\partial^\mu u_\mu\phi^4) \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
\nabla_\mu \mathcal{T}^{\mu\nu}_{total} &= \nabla_\mu [\mathcal{T}^{\mu\nu}_1 + \mathcal{T}^{\mu\nu}_2 + \mathcal{T}^{\mu\nu}_3 + \mathcal{T}^{\mu\nu}_4] = 0 \\
\nabla_\mu \mathcal{T}^{\mu\nu}_{total} &= [\frac{1}{\sqrt{-g}}(8\alpha\phi + \beta\phi^4)u^\nu + \frac{14}{3\sqrt{-g}}\partial_\mu\phi g^{\mu\nu} \\
&+ 4\Gamma^{\nu\mu}_{\mu}\frac{14}{3}\eta T\phi - \frac{16}{3\sqrt{-g}}\eta T(\partial_\mu\phi^4)g^{\mu\nu} \\
&- 4\Gamma^{\nu\mu}_{\mu}\frac{16}{3\sqrt{-g}}\eta T\phi^4]\partial^\mu u_\mu \\
&+ \frac{1}{\sqrt{-g}}(8\alpha\phi + \beta\phi^4)u^\mu(\partial_\mu u^\nu) = 0 \quad (17)
\end{aligned}
$$

From the rich structure of the conserve energy-momentum tensor equation, one can take some partial relation for further elaboration. For example, the conservation of energy (in special relativity frame)

$$\partial_\nu T^{0\nu} = \partial_0 T^{00} + \partial_1 T^{01} + \partial_2 T^{02} + \partial_3 T^{03} = 0 \tag{18}$$

Then, also the conservation of momentum. Based on these conservation laws, the bulk and shear viscosity distribution are currently in the progress of construction.